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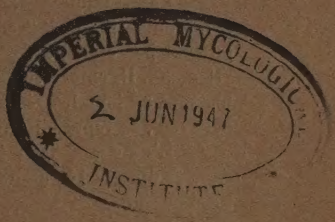

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MATHEMATICS

ON NILPOTENT ALGEBRAS AND p-GROUPS

By I. D. ADO

(Communicated by S. L. Soboleff, Member of the Academy, 27. IV. 1943)

In this paper a connexion between p-groups and nilpotent algebras over a field of classes modulo p is stated, where p is a prime number. This permits to prove some theorems concerning the structure of locally finite p-groups*, and to form examples for various kinds of groups. In particular, I have found an example of a p-group coinciding with its commutator subgroup.

An associative ring will be called weakly nilpotent, if all its elements are nilpotent. Further, if each finite set of elements of a ring generates a nilpotent subring, this ring will be called a locally nilpotent ring.

Let x be some element of a ring. A set of elements ξ which are of the form $\xi = a_0 x + x b_0 + a_1 x b_1 + \ldots + a_m x b_m$, where a_i, b_k are arbitrary elements of the ring, and m is an arbitrary number, forms an ideal which we denote by $N(x)$.

An element of a ring which gives zero after multiplication by any element of this ring will be called nilelement. If z is a nilelement, then $N(z)$ is the zero ideal, and conversely.

Lemma. *If N is a locally nilpotent ring, and x is one of its elements, then the ideal $N(x)$ does not contain x.*

For, if $x \subset N(x)$, then we should have

$$x = a_0 x + x b_0 + a_1 x b_1 + \ldots + a_k x b_k.$$

This is impossible, because, according to the definition of locally nilpotent rings, the elements $a_0, b_0, \ldots, a_k, b_k$ generate a nilpotent subring.

In the sequel linear algebras over a field P of classes modulo p will be considered, where p is a prime number. Their basis is assumed to be finite.

Let N denote a weakly nilpotent algebra over P. N permits to form a p-group in the following manner. We add to N the principal unity E and form all elements $E+u$, where u runs over N. These elements constitute a p-group with regard to multiplication. If, in fact, $u^s = 0$ and $p^k \geqslant s$, then $(E+u)^{p^k} = E + u^{p^k} = E(p)$. This group will be denoted by $p(N)$. An element of $p(N)$ arising from an element u of N will be denoted by $p(u)$. To every subalgebra of N there corresponds a subgroup of $p(N)$, and to every ideal N_1 of N there corresponds an invariant subgroup $p(N_1)$ of $p(N)$ which consists of elements $p(x)$, where $x \in N_1$.

If N is the algebra of classes modulo N_1, then the group $p(N)$ is iso-

* A group is called locally finite when each finite subset of its elements generates a finite group.

morphic with the factor group $p(N)/p(N_1)$. From the nilelements of N only elements of the centre of $p(N)$ can arise.

Now let us prove the reverse case and show how to form a nilpotent algebra by means of a p-group. First, we form a group algebra over a field P. The elements of this algebra have the form $n_1g_1+n_2g_2+\ldots+n_kg_k$, where g_i are elements of the group, and n_i are classes modulo p. The elements for which $n_1+n_2+\ldots+n_k\equiv 0(p)$ holds, constitute a subalgebra which will be denoted by R_0.

Lemma. *If G is a finite p-group, then the corresponding R_0 algebra is nilpotent.*

It is known that the sequence of successive commutator groups of a finite p-group terminates by the unity group (1). We shall prove the lemma by induction relative to the length of the sequence of commutator groups. In the first place the lemma is true for Abelian groups. For, if a_i are elements of an Abelian group, and k is an integer such that for all a_i $a_i^{p^k}=e$ holds (e is the unity of the group), then

$$(\sum n_ia_i)^{p^k}\equiv\sum n_ia_i^{p^k}=(\sum n_i)\,e\equiv 0(P).$$

Now we assume the lemma to be already proved for the commutator subgroup $C(G)$. The factor group of G relatively to $C(G)$ is Abelian. Therefore, if k is an integer such that for all g_i $g_i^{p^k}\in C(G)$ holds, then $(\sum n_ig_i)^{p^k}=\sum c_jg_j$, where the c_j lie in the subalgebra of R_0 generated by commutator elements. For $(\sum n_ig_i)^{p^k}$ is a sum of terms having the form $n_{i_1}n_{i_2}\ldots n_{i_\mu}g_{i_1}g_{i_2}\ldots g_{i_\mu}$ $(\mu=p^k)$. By joining the terms having the same composition, and by interchanging the factors g_i in a proper way, we can write the sum of these terms in the form $n_{i_1}\ldots n_{i_\mu}g_{i_1}\ldots g_{i_\mu}(c_1+c_2+\ldots+c_t)$, where $c_\nu\in C(G)$. The integer t is equal to the corresponding polynomial coefficient, and therefore is divisible by p.

Since, according to our inductive assumption, this subalgebra is nilpotent, a certain power of the element $\sum c_jg_j$ must vanish. Thus, we have proved that the algebra R_0 is weakly nilpotent. Since the basis of R_0 is finite, the lemma is proved.

If we consider an infinite, but locally finite p-group, the corresponding algebra is locally nilpotent.

By means of an algebra R_0 we can form a p-group $p(R_0)$ in a manner just indicated. This p-group, obviously, contains the given group as subgroup. In all following formulations the isomorphic groups are regarded as not different.

Principal Theorem. *Let G be a locally finite p-group. We can form a locally nilpotent algebra N over a field P such that the group $p(N)$ contains G as subgroup, and to each proper ideal S of the algebra N there corresponds an invariant subgroup $p(S)$ such that its intersection with G is different from the unity group.*

Let R_0 be an algebra formed by means of the group G in a way indicated above. In R_0 there exists a maximal ideal containing no elements of a given set, i. e. such an ideal that every ideal containing it must contain elements of the given set. Its existence follows from the fact that the smallest ideal, containing an increasing set of ideals which contain no elements of a given set, does not contain elements of this set either. It will be obvious that such a maximal ideal is, by no means, defined uniquely. Let G_0 be the set of the elements $x\in R_0$ for which $p(x)\in G$. We denote by N an algebra of congruence residue of R_0, when we take a maximal ideal not containing the elements of G_0 as modulus. Since R_0 is locally nilpotent, N is locally nilpotent also, and satisfies to the conditions of the theorem.

The theorem just proved reduces many problems concerning p-groups to the corresponding problems concerning nilpotent algebras. Thus, non-simplicity of a locally finite p-group follows from the existence of an ideal in the corres-

ponding locally nilpotent algebra. Malcev [2] has proved that non-simplicity is a consequence of a certain proposition of mathematical logic. However, the theorems proved below are not implied by the results obtained by Malcev.

Theorem 1. *Let G be a locally finite p-group. An arbitrary finite set of its elements being given, we can find its invariant subgroup not containing the elements of this set, unless they belong to the centre of G.*

Let N be a locally nilpotent algebra, which satisfies to the conditions of the principal theorem. Let $g_1, g_2, \ldots, g_n$ be a given finite set of the elements of G. Let $z_1, z_2, \ldots, z_n$ be their corresponding elements of the algebra N, i. e. $p(z_i)=g_i$. None of these elements is nilelement, as the elements g_i do not lie in the centre. It is easy to form an ideal H not containing z. At first, we form $N(z_1)$. If it contains another elements z_i, say z_2, then $N(z_2)$ contains neither z_1 nor z_2. Step by step, we form finally the required ideal H. The intersection $G \cap p(H)$ is different from unity, because H is different from the zero ideal, and does not contain the elements g_i since H does not contain z_i.

Theorem 2. *If a locally finite p-group G contains no centre, then it contains an infinite decreasing sequence of invariant subgroups.*

Let z be an element of the algebra N such that $p(z) \in G$. $N_1 = N(z)$ is no zero ideal, as z is no nilelement. Let an ideal N_i be already formed, and let $z_i \in N_i$, $p(z_i) \in G$. By putting $N_{i+1} = N(z_i)$ a sequence of ideals

$$N_1 \supset N_2 \supset \ldots \supset N_i \supset \ldots$$

arises to which a sequence of invariant subgroups $p(N_i) \cap G$ $(i=1, 2, \ldots)$ of G corresponds. Since $p(z_i)$ is contained in $p(N_i) \cap G$, but not in $p(N_{i+1}) \cap G$, each subsequent term of the latter sequence is a proper invariant subgroup of the foregoing term.

The existence of centre in a locally finite p-group whose decreasing sequences of subgroups are finite, has been proved by Tschernikow [3]. Theorem 2 shows that for the existence of a centre only the finiteness of decreasing sequences of invariant subgroups is sufficient. However, if a p-group is not simple, it may still coincide with its own commutator subgroup. The example given below shows that for each prime number p one can form a p-group coinciding with its commutator subgroup.

Let r denote the set of rational numbers in the interval $[0, 1]$ ordered in the natural way. We consider the unities $e_{\alpha\beta}$, where α and β run over the set r, and $\alpha < \beta$. Let $e_{\alpha\beta}$ be governed by the usual matric law of composition:

$$e_{\alpha\gamma} e_{\gamma\beta} = e_{\alpha\beta}, \quad e_{\alpha\beta} e_{\gamma\delta} = 0.$$

Further, we consider the algebra formed by these unities over the field of the classes modulo p. It is locally nilpotent, as every finite set of its elements is contained in a certain algebra isomorphic with a three-cornered matric algebra.

Now we adjoin the principal unity E and form the elements $E + e_{\alpha\beta}$. These elements generate a p-group, as has been shown above. We prove that every element $E + e_{\alpha\beta}$ is a commutator of certain properly chosen elements. Indeed, for arbitrary $\alpha, \beta\,(\alpha < \beta)$ we can find in r a number γ such that $\alpha < \gamma < \beta$. By putting $a = E + e_{\alpha\gamma}$, $b = E + e_{\gamma\beta}$, we obtain $a^{-1}b^{-1}ab = E + e_{\alpha\beta}$.

Thus, the p-group generated by the elements $E + e_{\alpha\beta}\,(\alpha, \beta \in r)$ coincides with its commutator subgroup.

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ASTRONOMIE

SUR UNE MÉTHODE POUR AUGMENTER LA CONVERGENCE DES SÉRIES TRIGONOMÉTRIQUES FONDAMENTALES POUR LA MÉCANIQUE CÉLESTE

Par M. F. SUBBOTIN

(Présenté par S. N. Bernstein, de l'Académie, le 26. VII. 1943)

1. Les premiers essais d'appliquer les méthodes analytiques au calcul des perturbations des petites planètes et des comètes ont mis en évidence la nécessité d'augmenter la convergence des séries trigonométriques qui sont employées dans ces méthodes. En laissant de côté les anomalies partielles de Hansen qui se sont montrées trop peu pratiques, nous n'avons pour atteindre ce but que deux transformations du développement

$$\Delta^{-1} \equiv (1-2\alpha\cos H+\alpha^2)^{-\frac{1}{2}} = \sum_{-\infty}^{+\infty} b_{1/2}^{(n)} \cos nH, \tag{1}$$

fondamental pour la Mécanique céleste.

La première de ces transformations est définie par les égalités

$$\Delta \sin S = \sin H, \qquad \Delta \cos S = \cos H - \alpha, \tag{2}$$

qui donnent

$$(1-\alpha^2)\Delta^{-1} = \alpha\cos S + \sqrt{1-\alpha^2\sin^2 S}\,; \tag{3}$$

il en résulte que les coefficients du développement

$$\Delta^{-1} = \sum_{-\infty}^{+\infty} C_n \cos nS \tag{4}$$

sont, pour des petites valeurs de α, de l'ordre de $(\alpha/2)^n$, tandis que les coefficients du développement (1) sont comparables à α^n.

Cette transformation, indiquée par Legendre (1), a été appliquée par Brendel au calcul des perturbations des petites planètes et même à la théorie des planètes principales (2, 3).

La deuxième transformation, étudiée avec soin par Gyldèn (4), consiste dans l'introduction d'une nouvelle variable T par les formules

$$H = \pi + 2\operatorname{am}(u, k); \qquad u = \frac{K}{\pi}T; \qquad k = \frac{2\sqrt{\alpha}}{1+\alpha}. \tag{5}$$

En vertu des relations connues

$$\frac{1+\alpha}{\Delta} = \frac{1}{\operatorname{dn} u} = \frac{\pi}{2k'K} + \frac{2\pi}{k'K}\sum_{n=1}^{\infty} \frac{(-q)^n}{1+q^{2n}}\cos nu, \tag{6}$$

$$q = e^{-\pi\frac{k'}{K}} = \frac{\alpha}{4} + 4\left(\frac{\alpha}{4}\right)^3 + 34\left(\frac{\alpha}{4}\right)^5 + \ldots \tag{7}$$

ceci donne pour Δ^{-1} le développement dont les coefficients décroissent (toujours pour des petites valeurs de α) comme $q^n \approx (\alpha/4)^n$.

Mme N. Samoilowa-Jachontowa avait introduit la variable T de Gyldèn dans les formules de Brendel et dans la méthode de Bohlin ([5-8]). Elle a fait aussi cette remarque intéressante que la variable T, proposée par Gyldèn, peut être regardée comme valeur approchée de la variable qui régularise les chocs simples dans le problème de trois corps où intervient le développement (1).

Dans ce qui suit, je donne une méthode très générale pour augmenter la convergence des séries trigonométriques qui représentent des fonctions analytiques. Cette méthode comprend comme un cas très particulier la transformation de Legendre et donne d'autres transformations algébriques assez simples, qui conduisent aux séries trigonométriques pratiquement aussi convergentes que les séries obtenues par la transformation de Gyldèn (5). La méthode générale suivie par moi montre en même temps à quoi tient le succès des transformations de Legendre et de Gyldèn.

2. Supposons que la série trigonométrique

$$f(H) = \sum_{-\infty}^{+\infty} a_k \cos kH, \qquad (a_{-k} = a_k) \tag{8}$$

représente une fonction analytique, en sorte que la série de Laurent correspondante

$$f(H) = \sum_{-\infty}^{+\infty} a_k z^k, \qquad z = e^{iH} \tag{9}$$

soit convergente dans un anneau circulaire $r < |z| < r^{-1}$. Le cas où l'on a

$$r = \lim_{k \to \infty} \sqrt[k]{|a_k|} = 0,$$

et, par conséquent, la convergence de la série (8) est particulièrement rapide, sera exclu dans ce qui suit. En supposant donc $0 < r < 1$, nous nous proposons de remplacer la série (8) par une autre à convergence plus rapide. Considérons pour cela la transformation

$$\prod_{n=1}^{m} \left(\frac{w - \alpha_n}{1 - \alpha_n w} \right) = \prod_{n=1}^{m} \left(\frac{z - \beta_n}{1 - \beta_n z} \right), \tag{10}$$

où $\alpha_1, \ldots, \alpha_m, \beta_1, \ldots, \beta_m$ sont des quantités réelles plus petites en valeur absolue que l'unité, qui fait correspondre le cercle $|z| = 1$ au cercle $|w| = 1$. En vertu de cette transformation nous aurons

$$f(H) = \sum_{-\infty}^{+\infty} A_k w^k, \tag{11}$$

avec $A_{-k} = A_k$.

Le domaine de convergence de cette dernière série sera donné par les inégalités de la forme $\rho_1 < |w| < \rho_1^{-1}$. Si, en choisissant convenablement les quantités α_n, β_n, nous avons pour ρ_1 une valeur ρ plus petite que r, la convergence de la série (11) et de la série trigonométrique correspondante

$$f(H) = \sum_{-\infty}^{+\infty} A_k \cos kS; \qquad w = e^{iS} \tag{12}$$

sera plus rapide que la convergence des séries (8) et (9). En effet, en désignant par ε une quantité positive arbitrairement petite, nous aurons

$$a_k = o((r+\varepsilon)^k); \qquad A_k = o((\rho+\varepsilon)^k).$$

3. Appliquons ces principes généraux au cas, où les points singuliers de la fonction (9) sont: 0, α, α^{-1}, ∞; $(0<\alpha<1)$. Telle est, par exemple, la fonction (1). Considérons d'abord les transformations (10) du premier ordre, c'est-à-dire celles pour lesquelles on a $m=1$. Sans restreindre la généralité on peut les ramener à la forme

$$w = \frac{z-\beta}{\beta z - 1}, \tag{13}$$

ne contenant qu'un seul paramètre β. Le domaine de convergence de la série transformée (12) sera ici le plus grand pour

$$\beta = \frac{\alpha}{1+\sqrt{1-\alpha^2}}. \tag{14}$$

Les points singuliers de la fonction (11) sont $\pm\beta$, $\pm\beta^{-1}$, et la série (1) pour laquelle $b_{l/2}^{(k)} = o((\alpha+\varepsilon)^k)$, sera remplacée par le développement

$$f(H) = \sum_{-\infty}^{+\infty} B_k \cos kS, \tag{15}$$

avec $B_k = o((\beta+\varepsilon)^k)$. La table qui donne les valeurs de β correspondantes aux valeurs indiquées de α, fait voir combien la convergence en sera améliorée.

$\alpha = 0.4$	0.5	0.6	0.7	0.8	0.9
$\beta = 0.209$	0.268	0.333	0.408	0.500	0.627
$\rho = 0.173$	0.222	0.276	0.338	0.414	0.522
$\rho' = 0.110$	0.141	0.175	0.214	0.263	0.332
$q = 0.104$	0.134	0.167	0.205	0.252	0.320

4. Parmi les transformations (10) du deuxième ordre ($m=2$) nous ne considérons pour le moment que celles de la forme

$$w^2 = \frac{(z-\beta_1)(z-\beta_2)}{(1-\beta_1 z)(1-\beta_2 z)}. \tag{16}$$

Soit d'abord $\beta_2 = 0$; nous aurons

$$w^2 = \frac{z(z-\beta_1)}{1-\beta_1 z} \tag{17}$$

en sorte que les points singuliers de la série (11) situés dans l'intérieur du cercle $|w|=1$ seront

$$0, \quad \pm\left(\frac{\alpha(\alpha-\beta_1)}{1-\alpha\beta_1}\right)^{\frac{1}{2}}, \quad \frac{\pm\beta_1}{1+\sqrt{1-\beta_1^2}}. \tag{18}$$

Il en résulte que pour $\beta_1 = \alpha$, c'est-à-dire pour la substitution

$$w^2 = \frac{z(z-\alpha)}{1-\alpha z}, \tag{19}$$

le domaine de convergence sera déterminé par les mêmes inégalités $\beta < |w| < \beta^{-1}$, que dans le cas précédent. Ce qui montre que la rapidité de convergence de la série transformée sera la même pour la transformation définie par les égalités (13), (14) et pour la transformation (19). Mais cette dernière conduit aux relations (4), donc elle est identique avec la transformation de Legendre.

Elle n'est pas du reste la plus avantageuse parmi les transformations (17).

En effet, désignons par β_1' la valeur de β_1 pour laquelle les modules de deux derniers des points singuliers (18) deviennent égaux, et posons

$$\rho = \frac{\beta_1'}{1 + \sqrt{1 - \beta_1'^2}}.$$

Alors, pour les coefficients de la série transformée correspondante

$$f(H) = \sum_{-\infty}^{+\infty} B_k' \cos kS \tag{20}$$

nous aurons évidemment $B_k' = o((\rho + \varepsilon)^k)$, où l'on a $\rho < \beta$ (voir le tableau).

5. Nous allons examiner maintenant la substitution (16). Pour avoir la meilleure convergence de la série transformée correspondante, nous devrons choisir les valeurs de β_1 et β_2 en sorte que les modules des points singuliers intérieurs au cercle $|w| = 1$ qui sont égaux à

$$\beta_1\beta_2|^{\frac{1}{2}}; \quad \left|\frac{\alpha-\beta_1}{1-\alpha\beta_1}\,\frac{\alpha-\beta_2}{1-\alpha\beta_2}\right|^{\frac{1}{2}}; \quad \left|\frac{(1-\beta_1\beta_2)^2 - \sqrt{(1-\beta_1^2)(1-\beta_2^2)}}{(\beta_1-\beta_2)(1-\beta_1\beta_2)}\right| \tag{21}$$

aient la limite supérieure la plus petite possible.

Ceci a lieu, comme il est facile de voir, lorsque ces trois quantités sont égales entre elles. En désignant par β_1', β_2' les valeurs correspondantes de β_1, β_2, par ρ' la valeur commune des quantités (21) pour $\beta_1 = \beta_1'$, $\beta_2 = \beta_2'$, et en écrivant la série transformée correspondante

$$f(H) = \sum_{-\infty}^{+\infty} B_k'' \cos kS,$$

nous aurons $B_k'' = o((\rho' + \varepsilon)^k)$.

La comparaison des valeurs de β, ρ et ρ' données dans le tableau montre bien dans quelle mesure augmente la rapidité de convergence du développement classique (1) par application des transformations (13), (17) et (16) avec le choix convenable des paramètres. La transformation de Legendre (19) conduit sous ce rapport au même résultat que la transformation (13).

6. Considérons maintenant la transformation de Gyldèn (5). En posant comme toujours $z = e^{iH} = e^{-i\,\mathrm{am}\,u}$; $w = e^{iT}$, on peut la présenter sous la forme

$$2\,\mathrm{sn}\,\frac{K \ln w}{\pi i} = \sqrt{z} + \frac{1}{\sqrt{z}}.$$

Donc, aux points singuliers 0 et ∞ elle fait correspondre respectivement les points q, q^3, q^5, ... et $-q$, $-q^3$, $-q^5$, ...

Il en résulte que le domaine de convergence de la série transformée sera toujours (et non seulement pour la fonction (6)) donné par les inégalités $q < w < q^{-1}$, et les coefficients de cette série peuvent être représentés par $o((q+\varepsilon)^k)$.

Si l'on compare, pour diverses valeurs de α, les valeurs correspondantes de q et de ρ', on voit que la substitution algébrique (16) donne pratiquement les mêmes résultats, au point de vue de la convergence, que la substitution transcendante employée par Gyldèn.

Institut de l'Astronomie théorique.
Académie des Sciences de l'URSS.

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PHYSICS

ELECTRIC VIBRATIONS IN PARTICLES OF A MASS-RADIATOR

By A. A. GLAGOLEVA-ARKADIEVA and A. P. KALUGINA

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The theory of action of an idealized mass-radiator (1), consisting of a mixture of a dielectric with small metallic globules uniformly distributed in a cubic system and acting in pairs like small Hertz vibrators, permits to compute the functions

$$W_v = \xi(k); \quad W_s = \eta(k), \tag{1}$$

where W_v is the energy stored in the vibrators contained in the unit volume of the mixture; W_s, the energy per unit area of one layer of globules in the mixture; k, the volume concentration of the mixture. Each of the functions (1) represented graphically gives a curve with a maximum; the first one has a maximum at a concentration mixture $k = k_{v\,\max}$ to which corresponds the maximum value of energy $W_{vk_{v\max}}$; the second one has a maximum at $k = k_{s\,\max}$ at which the maximum value of energy is equal to $W_{sk_{s\,\max}}$; let us denote the magnitude W_s with $k = k_{v\,\max}$ by $W_{sk_{v\,\max}}$.

For the samples of mixture differing from one another only by the size of particles the above values of energy will be the functions of the diameter of spheres D:

$$W_{vk_{v\,\max}} = \varphi(D); \; W_{sk_{v\,\max}} = \psi(D); \; W_{sk_{s\,\max}} = \vartheta(D). \tag{2}$$

In the figure the diagrams of functions $W_{vk_{v\,\max}} = \varphi(D)$ and $W_{sk_{v\,\max}} = \psi(D)$ are given; with an increase of D the first one decreases, while the second grows.

Sparking discharge between a pair of globules results in electrical vibrations in the latter exciting electromagnetic radiation. Formulae (1) and (2) deduced for the energy content in the case of a single charging operation of the vibrators in the mixture characterize the intensity of radiation according to the concentration of the mixture and the maximum intensity for the different samples of the mixture.

The experimental investigation of Kalugina (2,3) have established the relation between the intensity of radiation of the mass-radiator, proportional to the deflection of the galvanometer α, and the concentration k of the vibrational mass. The said relation $\alpha = f(k)$ is represented by a curve having with the concentration of mixture $k = k_{\max}$ a maximum $\alpha_{h\,\max}$. Measurements performed with several samples of mixture differing from one another by the size of particles only have shown the curves $\alpha = f(k)$ to have invariably a similar run. At a concentration of $k_{\max}$, varying according to the sample of mixture, the

maximum intensity of radiation $\alpha_{k_{max}}$ also varies for different samples of the mixture. The maximum intensity $\alpha_{k_{max}}$ was found to be the function of length of the particles l:

$$\alpha_{k_{max}} = F(l),$$

increasing with l.

In the experiments carried out by Kalugina ([2],[3]) the mass-radiator consisted of a mixture of aluminium filings and machine oil. By repeated sieving the samples were brought to consist of particles of nearly equal size, although a small admixture of coarser and finer particles was present. The length of a particle in each sample was determined by the arithmetic mean from measurements of length made for several dozens of particles. The average length l of the filings used by Kalugina for various samples of the vibrational mass was 0.062—0.094 cm. The measurements were made by means of a resonance thermocouple, resonating chiefly to the wave-length $\lambda = 1{,}2$ cm ([4]).

In case the ordinates are chosen properly, the experimental curve for small size particles $\alpha_{k_{max}} = F(l)$ coincides well with the theoretical curve $W_{sk_{v\,max}} = \psi(D)$ (for $l = 0.062$; 0.084 cm); at the transition to the large size particles the experimental curve rises sharply ([3]) (for $l = 0.089$; 0.094 cm), whereas the theoretical curve makes a more gentle ascent. Such a discrepancy of the curves is explained by us by the presence of a resonating detector taking part in the measurements. Indeed, in case the waves are generated by vibrators consisting of pairs of particles contained in the vibrational mass, the length of particles of such vibrators, tuned in resonance with the antenne of the thermocouple, may be computed according to the formula

$$\frac{\lambda}{a} = m \quad \text{or} \quad l = \frac{\lambda}{2m},$$

where $a = 2l$ is the length of the vibrator, λ is the length of waves radiated by it, and m the coefficient equal to 3.5—5.0 for the small Hertz vibrators ([5–7]), partly immersed in the dielectric medium—acetone, oil, kerosene or in the glass of the setting; taking into account the complete immersion into oil of the particles of mass-radiator, we put $m = 5.0$, which is the largest of all the above values of m; hence we obtain

$$l \approx \frac{1.2}{2 \times 5.0} \approx 0.12 \text{ cm}.$$

In the previous experiments of Kalugina the largest particles in a sample of vibrational mass had an average length $l = 0.094$ cm. The curve $\alpha_{k_{max}} = F(l)$ made an ascent, because the size of the particles gradually approached resonance with the antenne of the thermocouple. In order that the curve should pass through the maximum, the particles must exceed 0.12 cm in length.

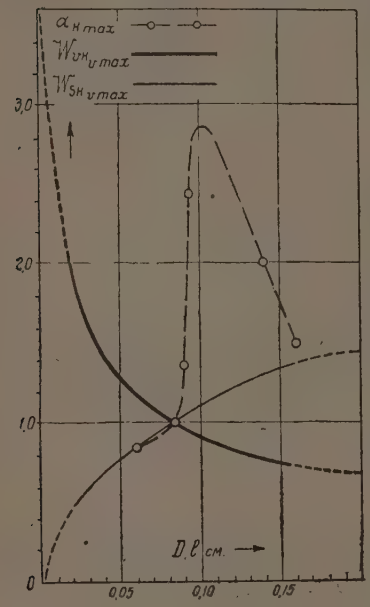
Measurements of $\alpha_{k_{max}} = F(l)$ were continued with samples of the vibrational mass containing particles 0.14—0.16 cm long. Measurements with samples of mixtures containing particles of a still greater length have been interrupted by the war circumstances. The results obtained are, however, sufficiently descriptive.

The results of measurements are represented in the figure, curve $\alpha_{k_{max}} = F(l)$; along the axis of abscissae the lengths l are plotted of different samples of the mixture; on the axis of the ordinates, the maximum deviations of the galvanometer $\alpha_{k_{max}}$ for each sample of the mixture with $k = k_{max}$. The curve has actually been found to have a maximum near $l = 0.103$ cm. On the left side this curve coincides with the curve $W_{sk_{v\,max}} = \psi(D)$; on the right it approaches declining this theoretical curve.

Curve $\alpha_{k_{max}} = F(l)$ represents the resonance curve of the thermocouple. The sharpness of the resonance, as shown by this curve, is less than the usual, in

so far as in each sample of the mixture only the mean length of particles is equal, which makes the curve broaden. In the case of an aperiodic wave detector the experimental curve must have a run similar to that of the theoretical one $W_{sk_{v\,max}}$.

The maximum of the experimental curve $\alpha_{k_{max}} = F(l)$, corresponing to the resonance between a pair of particles and the antenne of the thermocouple, makes it possible to calculate the coefficient m for small Hertz vibrators immersed in the vibrational mass. This maximum lies with $l = 0.103$ cm. In conformity with the above formulae we find



$$m = \frac{\lambda}{2l} = \frac{1.2}{2 \times 0.103} = 5.8,$$

where $2l$ is the length of the small vibrator.

The quantity $m = 5.8$ proves to be larger than 5.0 by 16%. This is probably due to the fact that the vibrators of the mass-radiator are surrounded with the vibrational mass which is a mixture of oil and metal particles, the dielectric coefficient ε of which is larger than the dielectric coefficient ε_0 of the oil, in which m may be equal to 5.0.

If in this particular complicated case the use of the formulae Mossotti-Klausius is allowed, then the above increase of the magnitude m may be fully justified quantitatively.

The run of the experimental curve $\alpha_{k\,max} = F(l)$ in its part remote from the resonance phenomena is analogous to that of curve $W_{sk_{v\,max}} = \psi(D)$ rather than to the run of curve $W_{vk_{v\,max}} = \varphi(D)$. This should evidently be taken to mean that the radiation of the mass-radiator proceeds chiefly from that area of the surface of the vibrational mass which is penetrated by discharge sparks.

The results obtained go, in the first place, to support the correctness of our interpretation of the operation of the mass-radiator; the short waves are actually generated by the electric vibrations arising in the particles of the vibrational mass; secondly, they prove that the theory of action of a mass-radiator, consisting of particles of a regular shape arranged in the dielectric medium according to a definite scheme, may be applied to a radiator in which the vibrational mass consists of moving particles of a complex and irregular shape suspended in the dielectric medium. This theory helps to explain the experimental data. Thirdly, the experimental curve (see figure) makes it possible to calculate the coefficient m for small vibrators immersed in the vibrational mass.

Maxwell Laboratory.
Moscow State University.
Academy of Sciences of the USSR.

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ON A NEW THEORY OF ELECTROMAGNETIC FIELD

By K. V. NIKOLSKY

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Bopp (1) and soon after him Podolsky (2) have formulated the principles of a new classical theory of electromagnetic field, exempt from various essential difficulties which are inherent in the hitherto known variants of the classical theory.

Bopp's theory is a variant of the theory of Mie, which implies the following choice of Lagrange's function L of the minimum action principle determining the field equations:

$$L = -\frac{1}{16\pi}\left\{f_{\alpha\beta}^2 + \frac{1}{\varkappa^2}\left(\frac{\partial f_{\alpha\beta}}{\partial x_\gamma}\right)^2\right\} + \frac{1}{c} s_\alpha \varphi_\alpha, \tag{1}$$

where $f_{\alpha\beta}$ is the antisymmetrical tensor of the electromagnetic field,

$$f_{\alpha\beta} = \frac{\partial \varphi_\beta}{\partial x_\alpha} - \frac{\partial \varphi_\alpha}{\partial x_\beta}, \tag{2}$$

s_α, relativistic charge vector; φ_α, potential; $1:\varkappa$, constant endowed with length dimensionality.

The minimum action principle

$$\delta \int L d\tau = 0 \tag{3}$$

corresponding to the variation of the independent variables $f_{\alpha\beta}$ leads to the following field equations:

$$\frac{\partial \Phi_{\alpha\beta}}{\partial x_\beta} = \frac{4\pi}{c} s_\alpha, \tag{4}$$

where

$$\Phi_{\alpha\beta} = -8\pi \frac{\partial L}{\partial f_{\alpha\beta}}, \qquad s_\alpha = c \frac{\partial L}{\partial \varphi_\alpha}. \tag{5}$$

The field $\Phi_{\alpha\beta}$ is thus determined through the quantities $f_{\alpha\beta}$ by the expression

$$\Phi_{\alpha\beta} = f_{\alpha\beta} - \frac{1}{\varkappa^2} \Box f_{\alpha\beta}. \tag{6}$$

The peculiarity of the electromagnetic field described by these equations consists in that it is determined by linear equations, i. e. satisfying the superposition principle and possessing, consequently, all the interference properties, having at the same time a limited character for the statical spherically symmetrical case, namely

$$\varphi = e_0 \frac{1 - e^{-\varkappa r}}{r}. \tag{7}$$

In the centre of the singularity the potential has thus a finite value $\varkappa e_0$, being not infinite as in Maxwell's case. Further, the energy density of the field of a point charge has the following form

$$W = \frac{1}{8\pi} \cdot \frac{e_0^2}{r^4} \{1 - (1 + 2\varkappa r + 2\varkappa^2 r^2) e^{-2\varkappa r}\}, \tag{8}$$

leading to a finite value of the field energy

$$\varepsilon_0 = \int w d\tau = \frac{1}{2} \varkappa_0 e_0^2. \tag{9}$$

These results of the new theory, differentiating it from the Maxwell-Lorentz theory which cannot lead to a finite mass of the electron, and from Born-Infeld's non-linear theory(4) which does not satisfy the superposition principle, suffice to require a careful examination of the principles of this theory.

Bopp's electromagnetic field can be divided in an relativistic invariant way: into a Maxwell field, i. e. a field satisfying equations

$$\frac{\partial \Phi_{\alpha\beta}}{\partial x_\beta} = \frac{4\pi}{c} s_\alpha, \quad \frac{\partial \Phi^*_\beta}{\partial x_\alpha} - \frac{\partial \Phi_\alpha}{\partial x_\beta} = \Phi_{\alpha\beta}, \tag{10}$$

and into a field of neutral particles considered by Yukawa(3), i. e. a field satisfying equations

$$\frac{\partial U_{\alpha\beta}}{\partial x_\beta} = \frac{4\pi}{c} s_\alpha - \varkappa U_\alpha, \quad \frac{\partial U_{\alpha\beta}}{\partial x_\alpha} - \frac{\partial U_\alpha}{\partial x_\beta} = \varkappa U_{\alpha\beta}. \tag{11}$$

Here

$$f_{\alpha\beta} = \Phi_{\alpha\beta} - U_{\alpha\beta}. \tag{12}$$

We wish to point out here a few fundamental questions which arise in connexion with the formulation of the principles of the new theory of the electron, and to suggest the way leading to their clarification.

Of fundamental importance here, obviously, is the question of the transition from the classical to the quantum approximation, i. e. of the method to account for the atomism of action. The scheme of calculation has been traced by Bopp, namely, the theory is formulated in the form of a Hamiltonian formalism, the quantization of which can be realized in the usual way.

A very interesting circumstance comes here to light, viz., that by the quantization of the field equations it proves possible to obtain, as a consequence of electromagnetic relations, the fundamental quantum relation, namely, Dirac's brackets, connecting the momentum p_l and the coordinate q_h of electron:

$$[q_h^{(\mu)}, p_l^{(\nu)}] = \delta_{hl}\delta_{\mu\nu}. \tag{13}$$

If the corresponding operators are defined by

$$q_h^{(\mu)} = \frac{1}{4\pi e_\mu} \int \varkappa_h \operatorname{div} D d\tau, \tag{14}$$

$$p_l^{(\nu)} = \frac{e}{c} a_l \ (r_\nu) = \sum_i \frac{e_i e_\nu}{2\pi c^2} \int v'_{i\mu} \psi_\omega \ (r_\nu - r'_i) \, e^{i\omega(t'-t)} \, d\omega \, dt', \tag{15}$$

where $\psi_\omega = \frac{1}{R}(e^{ikR} - e^{-k'R})$ for $k = \frac{\omega}{c}$

$$\varkappa' = \sqrt{\varkappa^2 - \frac{\omega^2}{c^2}} \text{ real and}$$

$$\psi = \frac{1}{R}(e^{ikR} - e^{-k'R}) \text{ for } k = \frac{\omega}{c},$$

$$k' = \frac{\omega^2}{c} \sqrt{1 - \frac{\varkappa^2 c^2}{\omega^2}} \text{ real}.$$

It should be remembered here that the attempt to obtain in a similar way Dirac's brackets in Born's electrodynamics fails to lead to a simple relation.

Although the classical theory of the field must be regarded as the limiting case of the quantum theory, the existence of the interference properties of the field, expressed by the linearity of the equations, must be considered as a quantum property expressing the quantum laws of the space—time coordination of the energy actions of the field.

The interference properties of the field are directly connected with the quantum laws of the motion and of the spin of the particles, described by this field. It is interesting to note that whereas the field can be split up in a relativistic invariant way into a Maxwellian field and into a Yukawa field of neutral particles, no splitting is possible for quantities quadratic with respect to the field, viz. for energy and momentum; this permits to avoid the classical divergencies inherent in the classical Maxwell-Lorentz field.

The interference properties of the field are directly related with the problem of the spin of particles, described by this field. With respect to this question Bopp says: «Die Stellung des Spins in unserer Theorie lässt sich ebenfalls vorläufig nur schwer ermessen». The spin of Bopp's field can be investigated by one of the following methods. On the one hand, making use of the linearity of the field equations, we can write them down in the operational form

$$\left[\frac{\partial}{\partial t}+k\right]\Omega=0, \qquad k=\alpha_\mu\frac{\partial}{\partial x_\mu}+\alpha_s\mu \qquad (16)$$

and define the operator L commuting with K

$$KL-LK=0 \qquad (17)$$

which represents infinitesimal rotations of the Lorentz group. By introducing the classical moment M

$$L=M+S, \qquad (18)$$

we obtain the spin S. On the other hand, the following fundamental properties should be noted. The spin is determined by the character of field transformation corresponding to transformations of the coordinates with respect to which the equations remain invariant. The invariant character of the splitting of Bopp's field into a field of Maxwell's type and one of Yukawa's type enables the following remarks (5) to be made: Two cases must be distinguished, that of particles with a zero rest mass $m=0$ and with a rest mass different from zero, $m\neq 0$. The first case is singular in that the field equations allow of gauge-invariance transformations which connect the states of the field in such a way that only two states prove possible, which cannot be transformed into each other by these transformations (two polarization states of the photon).

Considering in this case the problem of one body—a single photon, the spin is defined as a minimum value of the orbital moment of a particle. For a photon the square of the angular momentum $j(j+1)$ has a minimum value corresponding to $j=1$.

The case $m\neq 0$ to which Yukawa's particles belong is characterized by the following relations. We can choose in this case a reference system where the momentum of the particle vanishes and all the field components represented by plane waves depend on time only. With respect to rotational transformations the components of the field are transformed as a basic function of representation of the Lorentz' transformation group, i. e. are subjected to linear transformations, reducing to linear combinations of the initial functions. Each type of particles is described by the same irreducible representation of the rotation group. If its base consists of $2s+1$ functions, the independent components of the field, which are transformed jointly with the transformations of the coordinates, the number s is called the spin of the particles of a given type.

The particles described by the wave equations

$$\frac{\partial U_{ik}}{\partial x_k} - \varkappa^2 U_i = 0, \qquad U_{ik} = \frac{\partial U_k}{\partial x_i} - \frac{\partial U_i}{\partial x_k}$$

have a spin $s=1$.

The possibility of the relativistically invariant separation of Bopp's field into a Maxwell field and a field of the Yukawa type enables one to establish the analogy of Bopp's theory with a theory of particles, proposed by de Broglie(6), where the state of a particle is described with the 16 wave functions of the field. This 16-component field, as has been shown by Pauli, can be reduced to two irreducible fields—a field of particles with a spin $s=0$ and a field of particles with a spin $s=1$.

This analogy suggests the way to an analysis of Bopp's field. After writing down the field equations in the form (16), we reduce the matrices α_μ to their irreducible parts. In this connexion a great interest is presented by the investigation of the algebra of the matrices α_k, an investigation which is similar to that realized for the matrices β_k of the de Broglie's theory by Duffin (7).

After the expressions for the irreducible component parts of the field of an electron have been obtained, we can construct from them quadratic quantity to determine the energy relations holding for the field: the energy tensor, the momentum, the energy and the number of particles. Owing to the linearity of these equations, their quantization does not afford any difficulties.

The analysis of these questions is of fundamental importance, for it will throw light upon the relation between Bose-Einstein's statistics of the particles forming the field; and Fermi-Dirac's statistics of those particles which create this field and carry the electric charge. In the theory under consideration these particles can for the first time be constrcteud from the electromagnetic field, as shown by the finite value of the proper mass and by the possibility to obtain Dirac's brackets for the momentum and the coordinate of the particles.

P. N. Lebedev Physical Institute.
Academy of Sciences of the USSR.

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TECHNICAL PHYSICS

A METHOD OF PHOTOMETRIC INTERPOLATION WITH MIXED SPECTRA

By V. K. PROKOFIEV

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The method of photometric interpolation ordinarily employed (1) has certain shortcomings. For instance, in analysing for elements giving spectra with a small number of lines, lines rather far removed from one another have to be compared visually, which makes photometry more difficult. Furthermore, such lines may fall in regions with different values of the contrast factor (γ) of the photographic plate, which leads to errors in photometry. In this case Hartman's principle for correct photometry is violated, which requires that the pair of lines should lie in close proximity on the photographic plate and be of close wave-lengths.

Below we give a modification of the method of photometric interpolation which permits the photometry of always close lying pairs of lines of one and the same wave-length. It consists in combining the method of photometric interpolation with the method of mixed spectra.

Essentially the procedure is as follows. Using a stepped reducer throughout the work we first photograph spectrum 1 of some sample with a concentration C_1. Then the plate-holder with the photographic plate is shifted in the direction of dispersion and vertically by as much as 0.3 to 0.5 mm and another spectrum 2 of another sample with a concentration C_2 is photographed. Upon thus shifting the plate-holder, the second spectrum comes to be located in the intervals between the steps of the first spectrum, which is a considerable advantage when a continuous background is observed between the lines. Naturally, photography with mixed spectra may be employed for alloys giving spectra with a small number of lines. The method is not applicable to alloys like steel, unless spectrographs with a large dispersion are used.

Photometry of the lines is carried out in two operations. First, we visually, or with the aid of a microphotometer, measure the steps of the line λ_1 of the basic substance (the inner standard), comparing the line of the second spectrum with the respective line in the first spectrum. With this we obtain a photometric quantity $A=\lg I_2$, where I_2 is the intensity of the line in the second spectrum; the intensity I_1 of the same line in the first spectrum in accordance with the calibration of the reducer is put at 100.

Then the line λ_2 of the admixture being analysed is photographed. This line may lie quite far from λ_1; all that is necessary is that it form with line λ_1 a sufficiently homologous pair. Photometry gives the quantity $B=\lg i_2$, where i_2 is the intensity of the line λ_2 in the second spectrum; for the first spectrum the intensity i_1 of this line is taken as 100.

For the first sample we employ the relation $\lg \frac{i_1}{I_1}=K$, where K is an unknown quantity, but always constant for photographs of this sample in accordance with

the homology of the pair λ_1 λ_2. Then $\lg i_1 = 2.00 + K$. In this case the quantities I_2 and i_2 measured in the same units will be equal, respectively, to

$$\lg I_2 = A \text{ and } \lg i_2 = B + K.$$

We now turn to concentrations and make use of Sheibe's formula $\frac{i_2}{I_2} = ac\frac{b}{2}$ to obtain $\lg \frac{i_2}{I_2} = b \lg C_2 + a = B + K - A$, whence

$$b(\lg C_2 - \lg C_1) = B - A, \qquad (1)$$

i. e., the difference between the photometric quantities B and A gives the relative concentration of the second sample with respect to the first. If $C_2 = C_1$; then $B = A$. This makes it possible to control a certain limiting content of the admixture in the sample; to that end sample C_1 must be made a standard with a limit content of the given admixture. The concentration may be determined from formula (1) if the value of b is known for a given concentration range. The way to determine b is by taking photographs of the samples with extreme concentrations C_1 and C_2. On the basis of formula (1) a calibration curve can be drawn; in this case the point $C = C_1$ will serve for the origin of coordinates with a value of $B - A = 0$; the variable concentration C will be determined by the formula $b(\lg C - \lg C_1) = B - A$. The origin of coordinates is here a point of the curve.

To illustrate the method here described we present the results of measurements for Mg in aluminium alloys*. Photography conditions: the spectrograph used was one of the ISP-4 design produced by the State Optical Institute (SOI). Plate-holder 13×18, slit 0.08 mm, excitation by an activated a. c. arc at $I = 3$ amp., the upper electrode made of a pure aluminium. The photographs were taken without additional annealing with an exposure of 15″. A condenser lens placed before the slit gave an image of the arc on the collimator lens of the spectrograph. The pair of lines subjected to photometric measurement were Mg 2852—Al 2652. For measuring the steps an SOI photomicrometer was used and the data obtained were treated by the interpolation method. The mean for 4—6 steps was taken for each photograph. Each sample was photographed three times on three different plates. The concentrations of the samples are given in Table 1.

Table 1

Sample No.	Concentration of Mg (in %)	Sample No	Concentration of Mg (in %)
8	0.11	12	0.49
9	0.21	13	0.28
10	0.31	14	0.79
11	0.15	15	0.37

As a reference sample we took No. 10 which was photographed on every plate together with the spectrum of each of the samples tested, including No. 10 itself. Treatment of the results was carried out by means of the method proposed here, as well as by the ordinary method; in the latter case the lines of sample No. 10 on each spectrum were disregarded. In Table 2 the results are brought together.

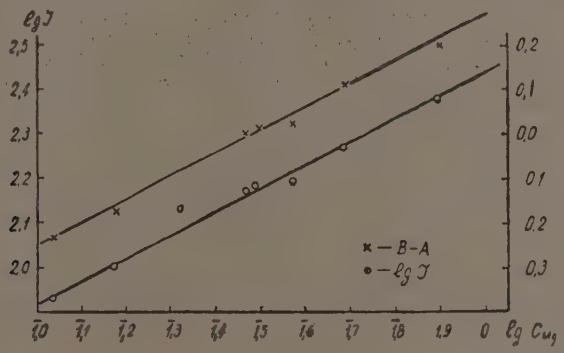
Here under $B - A$ are meant the photometric values obtained according to the method described above, and under $\lg I$; the photometric values obtained according to the ordinary method of interpolation of an analytical pair. For comparison with the values of $B - A$ in the last column there are given the differences

* The samples of the alloys were kindly supplied by K. A. Sukhenko. The photometry and measurements were carried out by laboratory assistant V. V. Prokofieva.

Table 2

Sample No.	$B-A$	$\lg I$	$\lg I_i - \lg I_{10}$
8	—0.23	1.93	—0.25
9	—0.07	2.13	—0.05
10	—0.02	2.18	0.00
11	—0.17	2.00	—0.18
12	0.11	2.27	+0.09
13	0.00	2.17	—0.01
14	0.20	2.38	+0.20
15	0.02	2.19	+0.01

between the value of lg I for each sample (third column) and the value of lg I for sample No. 10. As is evident from the table, the agreement between these latter values and those of $B-A$ is within the limits of 0.02. In other words, the values of $B-A$ do not introduce any considerable error, although here the error is increased approximately by 1.5 times ($\sqrt{2}$) as compared with direct measurements of lg I, owing to the fact that the difference is taken. In the figure are gi-



ven calibration curves plotted on the basis of the data of Table 1. We see that both curves are parallel to one another just as they should be. The linearity of the upper curve plotted with the values of $B-A$ is somewhat better, however. According to this graph, one may carry out an analysis by ever photographing sample No. 10 along with the unknown sample.

Conclusions. There has been proposed a method of photometric interpolation with mixed spectra. By means of it spectral lines lying closely together (as a result of which the error due to the non-homogeneity of the photographic plate is decreased) and of the same wave-length may be measured photometrically. The method also permits a comparison of lines far removed from one another in the spectrum, including those lying in regions with various values of the contrast factor of the photographic plate. For all elements of the alloy one may employ one line of the basic substance.

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GEOPHYSICS

EVAPORATION FROM WATER SURFACE AS DEPENDENT ON INFLOW OF RADIANT ENERGY

By M. SHVETZ

(Communicated by N. E. Kochin, Member of the Academy, 12. VI. 1943)

1. The present work is concerned with evaporation from an uncovered water surface ($z=0$); specific humidity on the latter is assumed equal to the humidity of saturated vapour, q_m:

$$q\,|_{z=0} = q_m. \tag{1}$$

Being a function of temperature T only, q_m can with a sufficient degree of accuracy be determined from Magnus' empirical formula:

$$q_m = 3.8 \times 10^{-3} \times 10^{\frac{7.45\,(T-273)}{T-38}} \tag{2}$$

The specific humidity $q(z, t)$ satisfies to the well-known equation of turbulent diffusion:

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial z} D \frac{\partial q}{\partial z}, \tag{3}$$

where D is the coefficient of vapour diffusion into air; t, time; z, vertical coordinate.

The temperature of air in a turbulent boundary layer is determined from the equation of heat conductivity:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \frac{\lambda}{c_p \rho} \frac{\partial T}{\partial z}, \tag{4}$$

where λ is the coefficient of heat conductivity; c_p, specific heat; ρ, air density.

Over the surface ($z=0$) the equation of heat balance takes place:

$$-\lambda \frac{\partial T}{\partial z} + \lambda_1 \frac{\partial T_1}{\partial z} + \sigma T^4 = A + S + r\rho D \frac{\partial q}{\partial z}, \tag{5}$$

where A is heat inflow (to unit surface) from long-wave radiation; S, heat inflow from short-wave radiation; r, latent heat of vapour formation; $r\rho D \frac{\partial q}{\partial z}$, heat expenditure on evaporation; σ, Stephan-Bolzmann's constant; λ_1 and T_1, coefficient of heat conductivity and temperature of the medium, where evaporation takes place. The inflow of heat $A+S$ is considered a known function of time.

2. Temperature T is determined from the equation

$$\frac{\partial T_1}{\partial t} = \frac{\partial}{\partial z} \frac{\lambda_1}{c_1 \rho_1} \frac{\partial T_1}{\partial z}. \qquad (6)$$

Besides the boundary condition (5), the following conditions also take place:

$$\left.\begin{aligned} &T(0, t) = T_1(0, t) \\ &T(\infty, t) < \infty, \quad T_1(-\infty, t) < \infty. \end{aligned}\right\} \qquad (7)$$

Let k and k_1 be the kinematic coefficients of air and water viscosity, respectively. Prandtl's number, both for evaporation $\left(\mathrm{Pr} = \frac{D}{k}\right)$ and for heat transmission $\left(\mathrm{Pr} = \frac{\lambda}{c\rho k}\right)$, is taken equal to unity; moreover, we put

$$k = bz^n \quad (0 < n < 1) \text{ and } k_1 = \text{const.}$$

Temperature variations being, as a rule, small compared to the mean (or initial) absolute temperature of the water surface ϑ, we shall assume, in order to linearize the problem, that $T = \vartheta + \tau$, $T_1 = \vartheta + \tau_1$.

Then instead of the equations (3), (4), (6) and of the boundary condition (1), (2), (5), (7) we shall have:

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial z} k \frac{\partial q}{\partial z}; \qquad (8) \qquad q(0, t) = Q_m + \nu \tau_1(0, t); \qquad (9) \qquad \frac{\partial \tau}{\partial t} = \frac{\partial}{\partial z} k \frac{\partial \tau}{\partial z}; \qquad (10)$$

$$\frac{\partial \tau_1}{\partial t} = \frac{\partial}{\partial z} k_1 \frac{\partial \tau_1}{\partial z}; \qquad (11) \qquad \tau(0, t) = \tau_1(0, t); \qquad (12)$$

$$\lambda \frac{\partial \tau}{\partial z} - \lambda_1 \frac{\partial \tau_1}{\partial z} - \mu \tau_1 = \Psi(t) + r\Phi(t), \qquad (13)$$

where $Q_m = 3.8 \times 10^{-3} \times 10^{\frac{7.45(\vartheta - 273)}{\vartheta - 38}}$; $\nu = Q_m \frac{7.45 \ln 10}{\vartheta - 38}$; $\Psi(t) = \sigma\vartheta^4 - A - S$;

$\Phi = k\rho \frac{\partial q}{\partial z}$; $\mu = 4\sigma\vartheta^3$.

3. The solution of (8), satisfying condition $k\rho \frac{\partial q}{\partial z}_{|z=0} = \Phi(t)$, has the shape:

$$q(z, t) = E \int_0^t \frac{\Phi(\alpha)}{(t-\alpha)^p} \exp\left[-\frac{p^2 z^{\frac{1}{p}}}{b(t-\alpha)} \right] d\alpha, \qquad (14)$$

where $E = \frac{p^{2p-1}}{\rho b^p \Gamma(p-1)}$, $p = \frac{1}{2-n}$, Γ being the Gamma function. Hence:

$$q(0, t) = E \int_0^t \frac{\Phi(\alpha)\, d\alpha}{(t-\alpha)^p}. \qquad (15)$$

It is quite obvious that

$$\tau(0, t) = \frac{E}{c_p} \int_0^t \frac{F(\alpha)\, d\alpha}{(t-\alpha)^p}; \qquad (16) \qquad \tau_1(0, t) = D \int_0^t \frac{\varphi(\alpha)\, d\alpha}{\sqrt{t-\alpha}}, \qquad (17)^*$$

where $F(t) = \lambda \frac{\partial \tau}{\partial z}(0, t)$; $\varphi(t) = -\lambda_1 \frac{\partial \tau_1}{\partial z}(0, t)$; $D = \frac{2}{c_1 \rho_1 \sqrt{k_1 \pi}}$.

According to (9) and (12) we have:

* The expression for τ_1 can be obtained from (16), putting there $b = k_1$, $p = 1/2$ and $\varphi(t) = F(t)$.

$$\int_0^t \frac{\Phi(\alpha)\,d\alpha}{(t-\alpha)^p} - \frac{Q_m}{E} = \frac{\nu}{c_p}\int_0^t \frac{F(\alpha)\,d\alpha}{(t-\alpha)^p}; \qquad (18) \qquad D\int_0^t \frac{\varphi(\alpha)\,d\alpha}{\sqrt{t-\alpha}} = \frac{E}{c_p}\int_0^t \frac{F(\alpha)\,d\alpha}{(t-\alpha)^p}. \qquad (19)$$

The equation of heat balance (13) may be represented in the form:

$$F(t)+\varphi(t)-\mu E\int_0^t \frac{F(\alpha)\,d\alpha}{(t-\alpha)^p} = \Psi(t)+r\Phi(t). \qquad (20)$$

Eliminating from (20) the functions F and φ by means of the correlations (18), (19) and of the equality $\int_\alpha^t (\beta-\alpha)^{-\nu}(t-\beta)^{-n}\,d\beta = (t-\alpha)^{1-\nu-n} \times$ $\times B(1-\nu, 1-n)$, where $B(x, y) = \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}$, for determining the rate of evaporation $\Phi(t)$ we obtain the integral equation:

$$\int_0^t \Phi(\alpha)(t-\alpha)^{-p}\left[a_0+a_1(t-\alpha)^{\frac{1}{2}}+a_2(t-\alpha)^{p-\frac{1}{2}}\right]d\alpha =$$

$$= \int_0^t \frac{\Psi(\alpha)\,d\alpha}{\sqrt{t-\alpha}} + b_0 + b_1 t^{1/2} + b_2 t^{p-\frac{1}{2}},$$

in which

$$a_0 = \frac{E}{D\nu}; \quad a_1 = -\frac{\mu E c_p}{\nu} B\left(1-p, \frac{1}{2}\right); \quad a_2 = \frac{c_p}{\nu} - r;$$

$$b_0 = \frac{Q_m}{D\nu}; \quad b_1 = -\frac{2\mu c_p}{\nu} Q_m; \quad b_2 = \frac{Q_m c_p}{E\nu\left(p-\frac{1}{2}\right) B\left(1-p, p-\frac{1}{2}\right)}.$$

On the contrary, eliminating from (20) the functions $\varphi(t)$ and $\Phi(t)$, we find the following integral equation to determine the flow of heat $F(t)$:

$$\int_0^t F(\alpha)(t-\alpha)^{\frac{1}{2}-2p}\left[\tilde{a}_0+\tilde{a}_1(t-\alpha)^{1/2}+\tilde{a}_2(t-\alpha)^{p-1/2}\right]d\alpha = \frac{rQ_m}{E} + \int_0^t \frac{\Psi(\alpha)\,d\alpha}{(t-\alpha)^p}$$

$$\left(\tilde{a}_0 = \frac{E}{Dc_p} B\left(1-p, \frac{1}{2}-p\right); \quad \tilde{a}_1 = -\mu E B(1-p, 1-p); \quad \tilde{a}_2 = 1-\frac{r\nu}{c_p}\right).$$

The solutions of these integral equations may be obtained by the method of Whittaker exposed in [1].

4. The problem of evaporation in a stationary air flow is solved in a quite similar way as the unstationary problem solved above. Indeed, let the wind velocity u be directed along the axis OX. Let us assume that near the water surface the distribution of velocity according to height obeys the hydrodynamic turbulent law $u = U_0 z^{1-n}$; the equation of diffusion will assume the shape:

$$U_0 z^{1-n}\frac{\partial q}{\partial x} = \frac{\partial}{\partial x} z^n \frac{\partial q}{\partial z}.$$

Passing to the new variable ζ, determined by $\zeta = z^{1/p}\left(p = \frac{1}{2-n}\right)$, we have:

$$\frac{\partial q}{\partial x} = \frac{1}{U_0 p^2}\frac{\partial}{\partial \zeta}\zeta^p \frac{\partial q}{\partial \zeta}.$$

Consequently, we have obtained an equation that coincides with equation (8).

Chief Geophysical Observatory.

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CHEMISTRY

CHEMICAL STABILITY OF Mg-Cu ALLOYS IN SULPHURIC ACID AND AMMONIUM CHLORIDE

By V. E. WAINSTEIN and E. E. WAINSTEIN

(Communicated by V. G. Chlopin, Member of the Academy, 21.V.1943)

In the present work the chemical stability of alloys of the system Mg-Cu in 0.1 *N* H_2SO_4 and a normal solution of NH_4Cl is investigated. The diagram of state of the system Mg-Cu, which has been studied time and again, is presented, according to Jones, in Fig. 1 and is characterized by the presence of two intermetallic compounds: $MgCu_2$ and Mg_2Cu. Thirteen samples of alloys in the range of compositions from pure Mg to 33.7% by weight of the latter were investigated. In Tables 1 and 2 are given the compositions of the samples studied and the losses in weight in H_2SO_4 and NH_4Cl, referred to a unit area of the cross-section of the latter. The alloys were prepared from commercial Mg trade-marked M-1 (99.7% Mg) and electrolytic copper. Fusion was carried out in a magnesite crucible in a gas furnace under a layer of carnallite. To obtain alloys with a comparatively low Cu content, use was made of an addition (50% Mg and 50% Cu) which was introduced into the melted copper. The copper was melted under a layer of charcoal and barium chloride. Melting of the alloys high in Cu was carried out in a graphite crucible, in which copper had been preliminarily melted several times. Corrosion tests preceded a metallographic study of the alloys. The entire surface of the samples (cylinders 15—20 mm high and 10—12 mm in diameter), except for one of the bases, was covered with a layer of paraffin after treatment with emery paper and fat removed in benzol. Losses in weight of the samples after they had remained for a period of two hours in a 0.1 *N* solution of H_2SO_4 or in a normal solution of NH_4Cl, expressed in mg/cm² hour, are given graphically in Figs. 2 and 3.

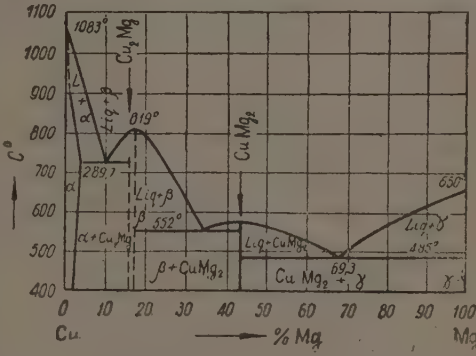

Fig. 1. Diagram of state of the system Mg-Cu (after Jones)

In the alloys treated with H_2SO_4, as the Mg content is decreased, a nearly linear drop is observed in the rate of corrosion curve right down to compositions ranging within 85—75% by weight of Mg. Then the curve becomes almost parallel to the axis of abscissae, declining slightly in the region high in $CuMg_2$. On this section the rate of chemical decomposition of the alloy is determined; evidently,

by the rate of disintegration of the highly disperse eutectic. Not until the isolated tiny inclusions of excessive Mg or round and needle-like excretions of Mg_2Cu seen on thin-sections pass into large contiguous areas, occupying (as e. g. in the case of the alloys with 45% Mg) nearly the whole field of a thin-section under the microscope, does the chemical stability of the alloys undergo considerable changes*. The linear portion of the curve extends right up to alloys with 45—46% by weight of Mg. And the content of the excessive compound Mg_2Cu in the alloy reaches 90—95%. Immediately after, a sharp decrease in the intensity of corrosion is observed, determined by smaller solubility of the compound as compared with the rate of disintegration of the eutectic. The alloys close in composition to the pure compound are distinguished by a minimum solubility. Upon further decrease in the Mg concentration, the rate of solution of the alloys mounts again,

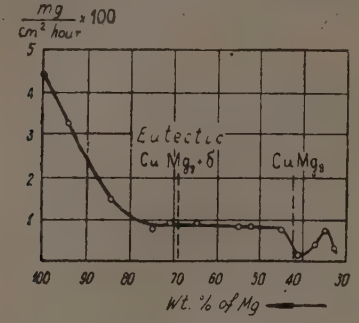

Fig. 2. Corrosion of Mg-Cu alloys in 0,1 *N* H_2SO_4

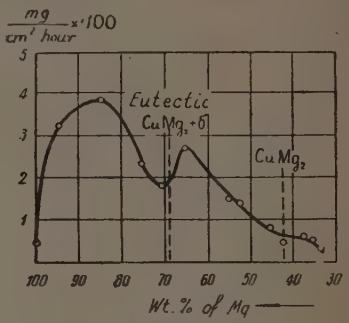

Fig. 3. Corrosion of Mg-Cu alloys in NH_4Cl

probably as a result of the appearance of a second compound, Mg_2Cu. Certain fluctuations in the course of the rate of chemical solution curve of the alloys, in particular fluctuations over the stretch of the linear portion, were observed depending on the degree of dispersion of the alloys. This had already been noted before by V. V. Skorchelletti (1) who had made a special study of the influence of the degree of dispersion of eutectic alloys in the systems Pb-Bi and Sn-Zn on the course of corrosion in them.

Somewhat different is the shape of the solution curve of the alloys in ammonium chloride.

The higher pH value in NH_4Cl as compared with sulphuric acid prevents the establishment of high current densities and makes the appearance of anode passivity of Mg little probable.

Table 1

No. of alloys	% Mg by weight	Specific loss in mg/cm² hour in H_2SO_4	No. of alloys	% Mg by weight	Specific loss in mg/cm² hour in H_2SO_4
1	100.0	435	8	52.7	82
2	94.6	322	9	45.0	77
3	84.8	140	10	41.8	15
4	74.8	83	11	37.5	43
5	70.4	89	12	35.1	72
6	65.2	90	13	33.7	28
7	55.1	80			

* The existence of «boundaries of permeability» of alloys, determined by a change in their structure in connexion with the nature of excretion of their components, was noted by V.V. Skorchelletti.

Table 2

No. of alloys	% Mg by weight	Specific loss in mg/cm² hour in NH_4Cl	No. of alloys	% Mg by weight	Specific loss in mg/cm² hour in NH_4Cl
1	100.0	473	8	52.7	135
2	94.6	314	9	45.0	84
3	84.8	380	10	41.8	45
4	74.8	236	11	37.5	56
5	70.4	181	12	35.1	48
6	65.2	270	13	33.7	22
7	55.1	145			

At small copper contents in the alloys large excretions of Mg cathodically protect the compound. This notwithstanding, the rate of destruction of the alloys increases as compared with the dissolution of pure Mg, on account of the microcells that are at work. Further; with a decrease in the quantity of Mg in the alloys, the compound begins to take an ever bigger part in the process of solution, the rate of solution of which in this medium is only two times as low as that of Mg (whereas the corresponding rates in 0.1 N H_2SO_4 differ tenfold). Upon reaching 85% by weight of Mg, the alloy acquires a «corrosion impermeable» structure, whereby the rate of destruction of the alloy is slowed down considerably. The rate of the corrosion curve continues to decline until compositions close to the eutectic are reached. The result of the appearance of the excessive compound in alloys lying outside of the eutectic is that in the field of compositions, stretching from the eutectic to 65% of Mg, a small increase in the rate of disintegration of the alloys in general is observed. Immediately after this the rate of corrosion curve begins to decline monotonously and continues to do so right up to compositions corresponding to the pure compound.

In conclusion we wish to express our thanks to V. V. Skorchelletti, who has suggested this work, and to I. V. Krotov, for valuable discussion of the results.

Institute of Mechanology.
Academy of Sciences of the USSR.

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CHEMISTRY

ON HETEROGENEOUS IRIDIUM SULPHITES OF THE TYPE OF CLAUS' SALT

By V. V. LEBEDINSKY and M. M. GURIN

(Communicated by V. G. Chlopin, Member of the Academy, 7.II.1943)

By acting with potassium bisulphite upon potassium chloroiridite, Claus [1] obtained the red crystals of a compound to which he assigned the formula:

$$(2KO \cdot SO_2 + 2KCl) + \left(2IrO + \left\{\begin{matrix} 2SO_2 \\ Cl \end{matrix}\right\}\right) + 12Aq.$$

The recalculation of Claus' analytical data gives the following formula of the salt:

$$K_4Ir(SO_3)_2Cl_3 \cdot 6H_2O.$$

According to Werner's assumption [2], all the six molecules of water that enter into the composition of the complex, belong to crystallization water; into the inner sphere of the complex enter two sulphite groups and three chlorine atoms:

$$K_4[Ir(SO_3)_2Cl_3] \cdot 6H_2O.$$

If we were to accept the formula of Werner, the problem as to the coordination number of iridium in the compound would yet remain unsolved. As well known, the sulphite group can be coordinately monovalent (univocal) or coordinately bivalent (bivocal). Thus, in the given case the coordination number is either five, or seven for iridium which always coordinates around itself six substituting groups and forms complexes according to Werner's octohedral scheme. Both alternatives seem scarcely probable. It is therefore but natural that students were in search for an explanation of the coordination structure of Claus' salt that would agree with the conceptions of Werner.

For example, Weinland [3], who thought it impossible to accept five as the coordination number for iridium, has suggested the idea that the salt of Claus is either an aquacomplex in which the sixth coordination place is occupied by water

$$K_4[Ir(SO_3)_2Cl_3 \cdot H_2O] \cdot 5H_2O,$$

or a polynuclear complex.

However, Weinland's assumption as to a possible existence of a polynuclear iridium complex can easily be refuted by experiment. Indeed, the simplest binuclear complex with two sulphite «bridges» should be a nona-ionic electrolyte:

$$\left[\begin{matrix} Cl_2 \cdot & \cdot SO_3 \cdot & \cdot Cl_2 \\ & Ir \quad Ir & \\ SO_2 \cdot & \cdot SO_3 \cdot & \cdot SO_2 \end{matrix}\right] K_6 \cdot 12H_2O.$$

As a matter of fact, measurements of molecular electrocenductivity of Claus. salt made by us gave the following results: with $v = 50^*$ ml $\mu = 427$ Ω; with $v = 1\,000$ ml $\mu = 485.3$ Ω; with $v = 2\,000$ ml $\mu = 516.9$ Ω (at 25° C).

These values approach closely the limits of Werner and Miolati for penta-ionic electrolytes; and it is just to the class of penta-ionic salts that the mononuclear complex should be referred. Let us note here that very close values of μ are also shown by the analogue of Claus' salt $Rb_3Na\,[Ir(SO_3)_2Cl_3]\cdot 6H_2O$, previously described by us (4).

Weinland's second assumption as to the possible existence of the aquasalt is not so easy to refute on an experimental way, for we possess as yet no sufficiently reliable analytical method to discriminate between inner-sphere and the crystallization water.

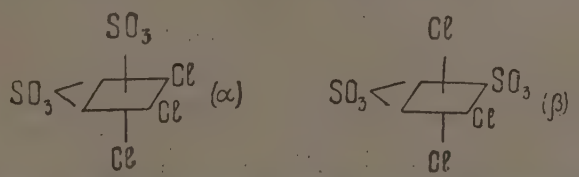
It should be noted, however, that in the case of Claus' salt and of its sodium rubidium analogue, as well as in the case of the zinc salt described below

$$Zn\,[Ir\,(SO_3)_2\,(NH_3)_3]_2 \cdot 8H_2O$$

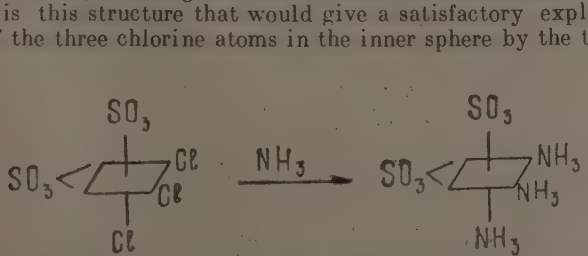
all the water is readily given off when heated to 100° C, whereby the salts suffer no changes that would point to a destruction of the inner sphere of the complex. From the standpoint of trans-influence it is also hardly possible that water be present in the inner sphere of Claus' salt (see below).

All these considerations make us now adopt for Claus' salt the formula suggested by Werner, $K_4[Ir(SO_3)_2Cl_3]\cdot 6H_2O$. At the same time we believe that the coordination number of iridium, six, is also valid in the present case. In an earlier paper by one of us (5) it has been shown on numerous examples of sulphito-complexes of cobalt, iridium, rhodium and platinum, described in literature, that, according to the conditions under which these complexes are formed, the sulphito-group occupies in their inner sphere either one or two coordination places. Now, if these conditions determine the coordination capacity of the sulphito-group that enters its composition, then it seems not impossible to choose such «average» conditions that, e. g., one of the two groups of the complex would occupy one coordination place, while the others would occupy two places.

If we extend this reasoning to the present case, then the structure of the salt of Claus and of its analogues can easily be explained within the limits of Werner's octohedron. Thus, the structure of the inner sphere of Claus' salt corresponds to one of the two possible isomers:

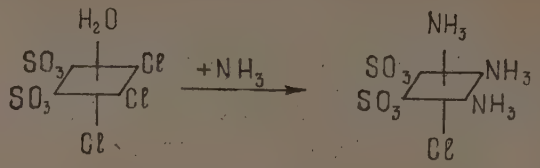


We think that Claus' salt is more likely to be an α-isomer («face» isomer) structure, in so far as, allowing for the increased trans-influence of the sulphito-group (4,6), it is this structure that would give a satisfactory explanation to the substitution of the three chlorine atoms in the inner sphere by the three ammonia molecules:



In case we were to admit for a moment that Claus' complex was an aquacomplex, the labilization of chlorine through an aqua-addend would appear scarcely possible.

Such an assumption would rather imply the formation, under the action of ammonia, of the disulphito-chloro-tetrammine salt, according to the following scheme of substitution in the inner sphere:



the trans-influence of water being much weaker than that of ammonia. As a matter of fact (see below), all the three chlorine atoms of Claus' salt are substituted by ammonia. Aside from the salt of Claus and its sodium-rubidium analogue described in our previous note, three more components were prepared, whose structure is of the type of Claus' salt.

I. Disulphito-triammine-iridate of zinc, $Zn[Ir(SO_3)_2(NH_3)_3]_2 \cdot 8H_2O$. The potassium salt of this series is formed when crystals of Claus' salt are treated with ammonium hydroxide taken in excess under heating over a water bath. Upon dissolution of the crystals the solution changes its colour gradually from red-orange (at the start of the run) to pale-yellow (end of the run). However, the potassium salt in solution could not be isolated because of its being readily soluble. When the solution is acted upon with alcohol, a viscous yellowish mass is separated that does not pass over into crystalline state. After the mass is solved in a minimum amount of water and subsequently acted upon with zinc nitrate, a white, fine crystalline precipitate of the zinc salt is separated. Its formation may be described by the following two reactions:

1. $K_4[Ir(SO_3)_2Cl_3] + 3\,NH_3 = K[Ir(SO_3)_2(NH_3)_3] + KCl$
2. $2\,K[Ir(SO_3)_2(NH_3)_3] + Zn(NO_3)_2 = Zn[Ir(SO_3)_2(NH_3)_3]_2 + KNO_3$

0.1434 g of salt gave 0.0540 g Ir
0.1526 g » » » 0.0578 g Ir
0.1434 g » » » 0.1273 g $BaSO_4$
0.1004 g » » » 0.0902 g $BaSO_4$
0.0952 g » » » 6.7 ml N_2 with 755 mm and at 20° C
0.1557 g » » » a loss of 0.0217 g H_2O at 100° C

For $Zn[Ir(SO_3)_2(NH_3)_3]_2 \cdot 8\,H_2O$

	Ir	S	N	H_2O
Calcd. in % . .	37.93	12.57	8.25	14.25
Found in % I . .	37.66	12.17	8.12	13.88
II . .	37.87	12.34	—	—

The salt gives off readily all the 8 molecules of water at 100° C and a further rise of temperature (up to 150°) leads to no decomposition of the salt. It is readily soluble in water.

II. Disulphito-trichloro-iridate of ammonium. Pentahydrate $(NH_4)_4[Ir(SO_3)_2Cl_3] \cdot 5H_2O$. 0.5 g of $(NH_4)_2CO_3$ were dissolved in 12 ml of water, saturated to an acid reaction, and the solution of NH_4HSO_3 obtained added 1 g of $(NH_4)_3IrCl_6 \cdot H_2O$. By gradually heating the solution, introduction of two sulphito-groups takes place, which may easily be traced after the change in the colour of the solution from olive-green to brown-red. Upon cooling of the solution filtered, separation of fine laminae coloured dark-red takes place. Separation of crystals may set in several hours after heating is stopped.

0.1012 g of salt gave 0.0314 g Ir
0.1012 g » » 0.0781 g $BaSO_4$
0.1012 g » » 0.0716 g AgCl

For $(NH_4)_4[Ir(SO_3)_2Cl_3]\cdot 5H_2O$

	Ir	S	Cl
Calcd. in % . .	31.05	10.39	17.04
Found in % . .	31.02	10.59	17.50

The content of water in this salt defies determination by the usual methods, because, if heated to 100° C, $(NH_4)_2SO_3$ evaporate together with water. Total loss of weight upon «drying» is 45—48%.

III. Disulphito-trichloro-iridate of ammonium. Trihydrate $(NH_4)_4[Ir(SO_3)_2Cl_3]\cdot 3H_2O$. Under the action of ammonium bisulphate (the concentration of which was 4 times that used in the preceding run), other conditions being similar to those used for the preparation of the salt described above, ammonium chloro-iridite, upon cooling of the solution, forms orange-red crystals whose composition was found from the following analyses:

0.1010 g of salt gave 0.0328 g Ir
0.1058 g » » 0.0348 g Ir
0.1010 g » » 0.0858 g $BaSO_4$
0.1058 g » » 0.0878 g $BaSO_4$
0.1064 g » » 0.0776 g AgCl

For $(NH_4)_4[Ir(SO_3)_2Cl_3]\cdot 3H_2O$

	Ir	S	Cl
Calcd. in % . . .	32.96	10.95	17.65
Found in % I . .	32.47	11.66	18.05
II . .	32.89	11.39	—

The geometrical arrangement of the addends in the inner sphere for the two last compounds probably corresponds to the configuration of the inner sphere of the salt of Claus (α-«facet»-isomer), the conditions of preparation of the potassium and the ammonium salt being very similar.

Institute of General and Inorganic Chemistry.
Academy of Sciences of the USSR.

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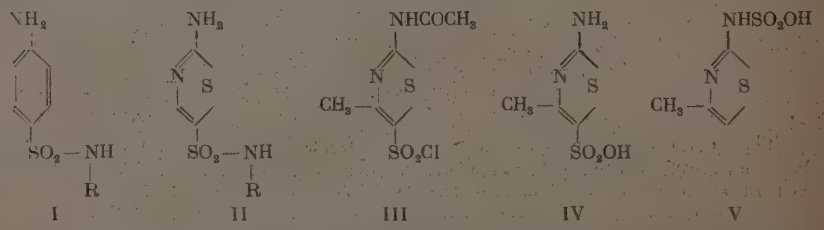
CHEMISTRY

ON SULPHOAMINOAMIDE COMPOUNDS OF THE THIAZOLE SERIES

By I. J. POSTOVSKY and T. S. BELAYA

(Communicated by A. E. Porai-Kochitz, Member of the Academy, 15. V. 1943)

Sulphoamide compounds, to which belong the well-known pharmaceutical preparations: streptocide (I, R—hydrogen), sulphapyridine (I, R—α-pyridyl), sulphathiazole (I, R—α-thiazolyl), are amides of sulphanilic acid. Taking into consideration the low toxicity of compounds containing the thiazole cycle, it seemed expedient to prepare similar sulphoamide compounds in which the sulphanilyl part of the molecule would be substituted by the 2-aminothiazole-5-sulphanyl group (II). The synthesis of such compounds was not likely to involve any difficulties, for 2-aminothiazole and 2-amino-4-methylthiazole behave in many reactions as aromatic amines (1):



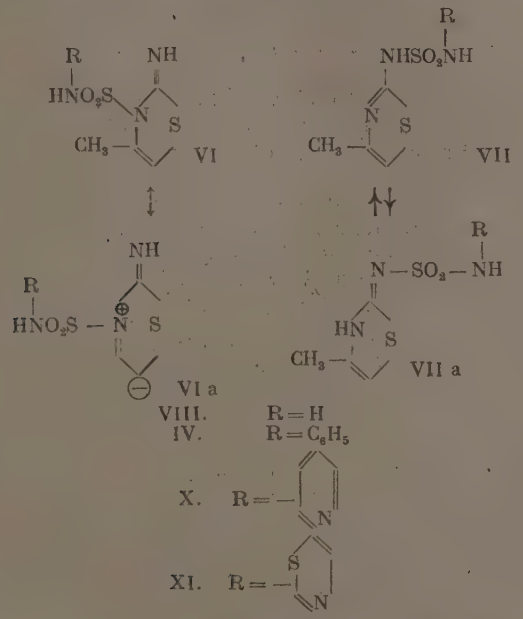
Similar to the reaction for the synthesis of compounds having structure I, for the synthesis of compounds II, acetylated aminothiazoles were treated with chlorosulphonic acid, and the sulphochlorides obtained were brought into reaction with bases, this yielding the corresponding amides (65—80%). After deacetylation with dilute hydrochloric acid free amides were obtained (yield 60—80%).

If the sulphochlorides and the amides obtained had the expected structure (II) and (III), then hydrolysis of the sulphochloride (III) should have yielded the 2-amino-4-methyl-5-sulphonic acid of thiazole (IV), that can be prepared according to Ochiai and Nagasawa(1) by the sulphonation of 2-amino-4-methylthiazole. As a matter of fact, instead of the sulphonic acid after hydrolysis we obtained the stable sulphamic acid (V), also described by these authors. Thus, it appears that the action of chlorosulphonic acids upon the acetylated 2-amino-4-methylthiazole leads to the formation, instead of the sulphochloride III, of the acetyl derivative of the chloride of sulphamic acid V* which reacts with amines under the formation of amides of sulphamic acid.

* The same sulphochloride was obtained also from sulphonic acid V. The acid was converted into its sodium salt, then acetylated with acetic anhydride and the acetyl derivative treated with PCl_5. As far as we know, this sulphochloride is the f i r s t example of a stable chloride of a sulphamic acid.

The new compounds VIII, IX, X and XI give with hydrochloric acid stable hydrochlorides and are rather resistant to acid media; whereas sulphonic acids of the aromatic series are easily decomposed in acid medium and are stable only in alkaline medium(²).

From the possible structures VI and VII we prefer structure VII, since substances of structure VI; owing to resonance structures where the interionized state VIa prevails; should be particularly liable to hydrolysis in acid media along the N—S bond [cf.(³)]. The susceptibility of sulphoaminoamides (VIII — XI) to alkaline solutions makes it possible as well to ascribe to them the imide form VII a.



The compounds VIII, IX, X, XI were tested for their chemotherapeutic properties. For the simplest compound, the sulphoaminoamide VIII; when tested *in vitro* by the method of agar cup worked out by V. P. Umanskaya*, high activity with respect to shaphylococci (*Staph. aureus pyogenes*, 1418) was observed. Sulphamic acid V also proved to be active; although to a much lesser degree compared with its amide VIII. As is well known, substitution of one hydrogen in sulphamide group of streptocide (I) through an α-pyridine or α-thiazole residue gives highly active preparations (sulphapyridine; sulphathiazole). In the case of the active compound VIII such a substitution leads to compounds X, XI which, contrary to expectations; showed no activity towards staphylococci in experiments *in vitro*.

Experimental. The chloride of sulphanilic acid was prepared by treating 2-acetylamino-4-methylthiazole with chlorosulphonic acid taken in excess (4—10 moles per 1 mole of the acetylamine). The sulphochloride was crystallized from dichloroethane. M. p. 156—157° (found %: S; 25.28; Cl: 14.16. Calcd. for $C_6H_7O_3N_2S_2Cl$ %: S, 25.18; Cl, 13.93).

The acetyl derivative of compound VIII was prepared through the action of gaseous ammonia upon the sulphochloride; the rest of compounds were prepared through condensation of the sulphochloride with corresponding amines in a solution of a double amount of pyridine. As an example, the description of the syn-

* Our thanks are due to V. P. Umanskaya for having kindly tested these preparations.

thesis of 2-[4-methylthiazole-2-sulphoaminoamide]-pyridine (compound X) is given below.

Synthesis of the Acetyl Product. In a flask fitted with a reflux condenser and a stirrer, 9.4 g of aminopyridine were dissolved in 19 cm³ of dry pyridine. Under stirring the solution was gradually added 25.4 g of the sulphochloride at a temperature of 35.40°. After the addition of the sulphochloride the mixture was heated to 90° and maintained at this temperature, under stirring, for 1 hour. Then, at 40°; 20 cm³ of water were gradually added and after two hours the yellowish, powdery precipitate was filtered off, washed first with water until the filtrate became colourless; then with 3% hydrochloric acid, and finally with water again. Yield of the dry product 24.9 g (80% of the theoretical). M. p. 225—226°. After crystallization from a 15-fold amount of alcohol, m. p. 226°.

Hydrolysis of the Acetyl Product. To 20 g of the acetyl product in a flask were added 60 cm³ of 15% hydrochloric acid and the mixture refluxed for 1 hour on a steam bath. The content of the flask passed rapidly into solution. To the hot solution 12 cm³ of water were added and the still warm solution was neutralized with dry $NaHCO_3$. After cooling the precipitate was filtered off, washed with water and dried on a water bath. Yield 12 g (78% of the theoretical amount). M. p. 209—210°. After crystallization from 50% alcohol the melting point underwent no changes.

Some constants and analytical data are tabulated below.

Product	Formula	Crystallized from	Shape of crystal under microscope	Melting point °C	Found N, %	Calculated N, %
VIIIAc*	$C_6H_9O_3N_3S_2$	water	oblique-angular plates	225 — 226	18.05	17.86
VIII	$C_4H_7O_2N_3S_2$	water	hexagonal prisms	168 — 169	22.10	21.75
IXAc	$C_{12}H_{13}O_3N_3S_2$	alcohol	rhombs	197 — 198	13.61	13.50
IX	$C_{10}H_{11}O_2N_3S_2$	50% alcohol	flat rhombs	139 — 140	15.70	15.60
XAc	$C_{11}H_{12}O_3N_4S_2$	alcohol	rhombic plates	225 — 226	17.97	17.94
X	$C_9H_{10}O_2N_4S_2$	50% alcohol	hexagonal tables	209 — 210	20.67	20.74
XIAc	$C_9H_{10}O_3N_4S_3$	glacial acetic acid	rectangular plates	236 — 227	17.69	17.61
XI	$C_7H_8O_3N_4S_3$	50% alcohol	rectangular rods	215 — 216	20.24	20.28

*Ac—acetyl derivative.

Thus, sulphoaminoamide compounds being a new type of sulphoamide preparation of the thiazole series (VIII—XI) have been synthesized.

Kirov Ural Industrial Institute.
Ural Branch of the S. Ordjonikidze
All-Union Chemical Pharmaceutical Institute.

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GEOCHEMISTRY

DATA ON TRACE ELEMENTS CONTAINED IN KOK-SAGHYZ

By S. A. BOROVICK, G. G. BERGMANN and T. F. BOROVICK-ROMANOVA

(Communicated by V. G. Chlopin, Member of the Academy, 4.V. 1943)

Considerable interest that centres about kok-saghyz as a source of raw material for rubber industry calls for a detailed investigation of this plant. No data on its content of rare and dispersed elements have as yet been produced. To fill this gap is the aim of the present paper which gives a possibly complete information on the subject. Spectrum analysis in the form developed in this laboratory allows a very accurate estimation of a series of elements: Be; Sb, Pb, Cu; Zn, Cd, Ni, Co, Zr, Mo, Ga, Cr, V, Ba; Sr; Ti and others. The determination has often been made by us, according to this method, of these elements in vegetable and animal organisms. The material taken for analysis was preliminarily reduced to ash. In Table 1 we give the list of the material analysed and the data of their ash content.

Table 1

Content of Ash in Kok-Saghyz Specimens

Nos.	Specimen	Origin	Ash content
1	Kok-saghyz seeds	Region of Kazan	5.5
2	» »	Region of Kursk	6.25
3	» »	» » »	6.33
4	» »	Province of Sumy	5.58
5	Roots, annual	District of Chistopol	4.66
6	Roots, biennial	» » »	8.0
7	Roots, annual	Kazan	6.73
8	Roots, biennial	»	8.33
9	Leaves, annual	District of Chistopol	16.66

For brevity sake, the ordinal number of this table will be used instead of the names of the respective specimens.

In taking spectrograms the ash was introduced into the recess of the lower electrode of an a. c. electric arc. The material was added every fifteen minutes. The current strength in the arc was about 10 A; 220 V. From the relative brightness of the lines the following conclusions could be drawn on the comparative concentration of a given element. In the seeds of four kok-saghyz specimens the following trace elements were discovered: Pb, Sn, Cu, Zn, Co, Ni, Mo, Ga, Cr, Sr, Ti. In the case of Pb, Cu, Zn, Ga and Cr no appreciable fluctuations of the concentration were observed. No tin line could be detected in the first and second specimens, but as the tin line in the fourth and third specimens was barely

visible, it may be assumed that the concentration of tin in these samples was close to the limit; beyond which it is undetectable. The same can be said of Co which showed no lines in the first specimen, and of Sr which was not detected in the second. As notable departures, one may consider the increased concentration of Ni in the second specimen and the low content of Mo in the third. With the exception of these two elements, all the rest showed very similar concentrations in the seed of different specimens of kok-saghyz; such as are usual for other plants.

The analysis data for roots of two kok-saghyz specimens showed a difference in the concentration of Ni and Zr. Namely, the content of these elements was notably high in the 5th specimen. In the other elements there was no difference at all, if only a very slight one. It is worthy of note that the content of trace elements in biennial roots was considerably higher than in annual ones. Especially high was the accumulation of Ni, Zr, Ga; Ti, Pb and Sr.

Particular attention deserves zirconium. This element is altogether absent from the list of the trace elements usually encountered in the organisms of plants and animals. In our analyses its presence was stated in both the root and leaf of kok-saghyz (specimens 5 and 9).

Table 2

Results of Analysis of Kok-Saghyz Specimens

Element	Seeds				Roots				Leaves
					Annual	Biennial	Annual	Biennial	
	Kazan	Kursk	Kursk	Sumy	Chistopol		Kazan		Chistopol
Pb	α	α	α	α	—	α	β	α	β
Sn	—	—	γ=	γ=	—	γ=	γ=	γ=	γ=
Cu	+	+	+	+	β+	α+	α+	α+	β+
Zn	+	++	++	++	γ=	α+	α+	α+	β+
Co	γ	γ	γ	—	—	—	—	—	—
Ni	α	+	α	α	α	α	α	+	γ
Zr	—	—	—	—	α	α	α	+	γ
Mo	α+	α+	β	α	γ	γ+	γ+	γ=	β
Ga	β+	β	β	β	γ=	α	γ+	α	β
Cr	γ	γ=	γ=	γ	γ	β	γ	γ+	β
V	—	—	—	—	—	γ=	γ=	α	γ=
Ba	—	—	—	γ	—	α+	α	γ+	α
Sr	γ	γ	—	γ+	β	+	β	I=	+
Al	α	α	α+	α+	α+	α+	α+	α	α
Mn	α	α	α+	α+	α+	α+	α+	++	α
Ti	γ	γ=	γ=	γ	α	+	++	++	α
Si	++	II	II	II	I	I	II	II	II

In Table 2 an arbitrary notation is used for appraising the line brightness of the analysed specimens: γ=, negligible traces of lines; γ, traces of lines; β, very weak lines; α, weak lines; +, medium lines; + +, above medium lines; II, strong lines; I, very strong lines. Plus indicates the strengthening of the brightness of the line; dash denotes the absence of lines of the respective element.

The data of the table refer only to trace elements.

W. I. Vernadsky Laboratory of Geochemical Problems,
Academy of Sciences of the USSR

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GEOLOGY

FACIOGRAMS—A NEW METHOD OF GRAPHIC REPRESENTATION OF STRATIGRAPHIC SECTIONS

By A. G. VOLOGDIN, Corresponding Member of the Academy

Graphic representations of the sections of sedimentary, sometimes also of volcanogene and magmatic rocks, as universally adopted in the shape of stratigraphic columns, mostly bear the character of a final fixation of the conclusions drawn by the investigator on the geological history of the region concerned.

Sometimes investigators make use of serial stratigraphic columns to solve the problems of correlation of the sections and to arrive at conclusions of a paleogeographical character. Therefore, the quality of the graphic representation of the columns which become the initial material, is of no minor importance. Usually, the investigator has recourse to different combinations of hatched symbols—straight and oblique bars, «crossfeet», etc.—which are inevitably of noticeable size and are usually an encumbrance when thin layers are to be represented. In the latter case some or other misrepresentations are artificially introduced into the section in the shape of exaggerated thicknesses of the strata, omitting certain bands and layers, etc. These admissions disturb the true picture of sedimentation in the region under consideration and allow no detailed correlation to be made between sections of remote regions.

In connexion with a treatment of certain materials on the stratigraphy of a portion of the Russian platform, the author was obliged to turn his attention to the above problem. At the same time he succeeded in working out and practically testing a new principle of graphic representation of stratigraphical and lithological sections.

The process of sedimentation at the bottom of any, in particular, of an epicontinental water basin, ancient or modern, is dependent in every point upon a number of factors. In a general case, when no lateral transfer of water masses is supposed to take place, coarse clastic material (rubble and pebbles) is formed and deposited in the zone of littoral surface at any particular moment, as a result of tidal wave activity; this clastic material, while being partly transferred and sorted by the ebbing waves; is carried on deep into the basin from the littoral zone, where, in accordance with a gradual weakening of the influence of wind waves and a gradual deepening of the bottom, an increasingly fine-grained material is deposited. The pebbles within the beach zone are gradually replaced by fine-grained sands, then by arenaceous slimes; fine slimes; and these, in the open sea, become more and more mixed with a carbonaceous sediment of a hydrochemical or biogenic origin; later on, the latter becomes the only type of forming sediment; undergoing sometimes; as a result of the influence of the mineralized water of the basin; certain alterations in chemical composition.

In this picture of sediment distribution roughly sketched by the author certain local anomalies may be found depending on the intricacy of configuration

of the shore-line, the relief of the bottom, the currents and the activity of rock-forming organisms. At the same time the carbonaceous sediments are deposited frequently in association or in the vicinity of clastic terrigene material. With an intensive development of vertical movements the lithological alteration of the sediments must be more sharp; rocks of transitional facies sometimes being omitted, and an occasional development of a continental environment being observed. It is, therefore; often very important to try and reestablish the geological life of the ancient basin in a stratigraphical-lithological column.

We are able; therefore; to assume a different, namely a faciological principle of designation of the sediments, springing from their connexion with facially different zones; instead of the customary hatched lithological symbols used to construct a stratigraphical column. This way we shall obtain a more advantageous material to solve paleogeographical problems, to trace geotectonical processes in space, etc. In diagrams we may graphically estimate the sea or shore origin of a particular deposit, i. e. the lithological composition of rocks formed in an environment of different facies—from littoral to open-sea facies.

Such an estimation may be made in terms of arbitrary points, whereby a representative faciological characteristic of the profile is obtained. The character of diagenetic alterations of rocks is here not to be taken into account because these are of secondary nature.

The author used for his graphical representations the following subdivision of sediments:

I. Marine Formation. 0—Littoral rapid and shallow-water sediments: rubble, pebbles, breccia, pebble-bed, coarse sand; cemented or not cemented (vertical line). 1—Medium- and fine-grained sands and sandstones. 2—Sands and argillaceous sandstones, loam. 3—Arenaceous clays, sandy loam. 4—Fatty clays. 5—Marly clays. 6—Arenaceous or argillaceous marls. 7—Normal marls. 8—Marly or arenaceous limestones. 9—Limestones. 10—Dolomites.

II. Continental-Lagoon Formation. 0—Continental environment without sedimentation. Symbols omitted or line of break. 1—Gypsum-anhydrite sediments in primary deposits. 2—Sea salts—primary deposits. 3—Aeolian deposits, etc.

For the time being no quantitative characteristics of the chemical composition of the rocks is introduced into this scale, as we usually have to deal with a macroscopic description of sections made by field geologists. Later on, with the gradual improvement of methods for rock determination, their composition, as expressed by terms of marinity, may be appraised more exactly, e. g. from their CO_2 content. Thus, we plot (on plotting paper) the rocks of marine facies to the right from a certain vertical zero-line on an arbitrary point scale. For instance, one point of marinity is equal to 5 mm. The thickness of the deposits is estimated in descending order or backwards, in a vertical direction on the same scale, e. g. 1 : 500, 1 : 200.

The continental or lagoon sediments are plotted to the left from the zero-line. The complex hydrochemical and clastic terrigene or carbonate sediments might be plotted on both sides of the zero-line; inasmuch as a corresponding successive or parallel drawing together of facies took place. As a result of plotting the data according to the textual description of the section, we obtain a broken faciographic line, the area between this and the zero-line being, for the sake of clearer demonstration, filled in with Indian ink; only for the thicker strata the numbers are noted from the description of the section of the plotted series.

As an example may serve the section of the deposits of the Ufa stage of the Lower Permian along the Lesnaya Sheshma river described by V. G. Sobolev and G. Raspopov in 1932 near the village of Petropavlovka in the south-eastern part of Tataria. Going in the downward direction we have:

9. Yellowish-brown loam	13.34 m
8. Grey clay	2.00 »
7. Grey limestone	0.50 »

6. Grey limestone with small brown spots	2,91 m
5. Argillaceous limestone with fauna	0.30 »
4. Grey slightly calcareous clay	8.02 »
3. Grey marl with vegetable remains	0.10 »
2. Brownish-grey bituminous sandstone	6.84 »
1. Bluish-grey pyritized clay	1.00 »

The adaptation of these data to a graphical representation is shown on the accompanying Fig. 1 on a 1 : 500 scale. A very peculiar figure is obtained, clearly demonstrating the lithological peculiarity of the series. While analysing the observed alternation of sediments in time, we find in this case an indubitable connexion between the sediments and the vertical movements of the bottom of the basin: first, a tendency to rise and become shallow (strata 1 and 2), then to deepen (strata 3—6), then again to rise (strata 8—9). If the authors of the description had noted the gradual nature of the transition of the rocks with decreasing marinity of the sediments (strata 1—2, strata 6—9), we could readily dispense with the idea of upheavals and explain the alteration of rocks to littorality by a gradual filling in of the basin. Thus, the faciogram

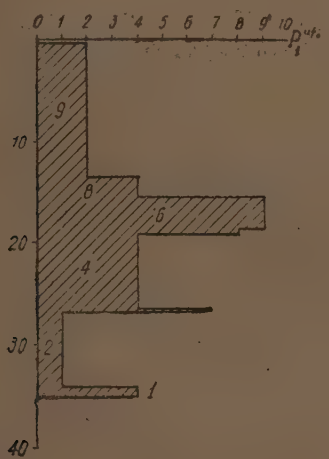

Fig. 1. Lesnaya Sheshma river, Petropavlovka village (after V. Sobolev and G. Raspopov, 1932)

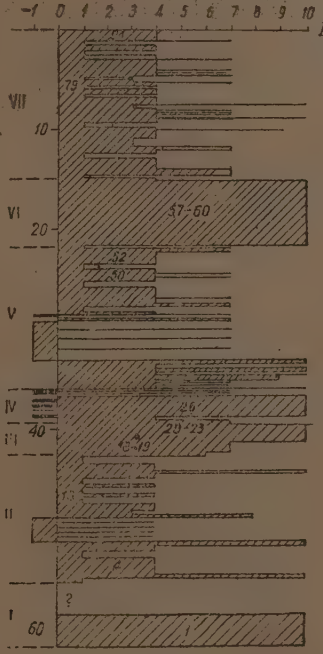

Fig. 2. Laishevo—Rybnaya Sloboda (after V. Butrov, 1932)

of the series reveals the mechanism of tectonical movements synchronous to the sediments and the close connexion existing between the accumulation of sediments and geotectonical processes.

A more complicated example may be given in the shape of a faciogram of the Conchifer substage of the Upper Permian. The section is described by the geologist V. Butrov in 1932 in the area of Laishevo and Rybnaya Sloboda along the Kama river (Fig. 2). On this graph prepared on the same scale (1 : 500) the descriptions are used of 94 horizons of the substage subdivided into the well-known series: I—laminated stone, II—podboi, III—grey stone, IV—shikhany, V—gaize, VI—podloozhnik, VII—transitional beds.

A correlation of sections of coeval sediments in different regions from faciograms clearly shows the degree of spatial stability of the facies and very frequently allows, even with a high variability of the sediments along the strike, reliably to correlate coeval strata from the similarity of character of sedimentation pro-

cesses. This similarity depends, evidently, on the unity of the geotectonical process in the region under consideration, and this may be clearly seen on the faciograms.

The geochemical data may be plotted as conventional symbols on the margins of the graph, at a corresponding stratigraphical level; the data showing the age may be also given as conventional indices.

Thus, the method of plotting faciograms, proposed by the author instead of the universally adopted hatched stratigraphic columns, possesses the following advantages:

a) The technics of faciogram correlation is much simpler, since uninterrupted covering with Indian ink is applied instead of complicated symbols.

b) The graph demonstrates the character of sediment alternation in connexion with the macro- and micro-oscillations within the basin.

c) The faciograms permit the marking horizons to be easily isolated for structural mapping and prospecting work and to trace them even under conditions of facial alterations from their gradual change in definite directions.

d) In cases of sharp differences in the lithological composition of coeval sediments in different regions, only the character of a faciographical line, reflecting the stages of geotectonic life in the region of the basin, may be preserved in order to correlate the sections. From the faciogram it is easy to plot a curve of tectonic movements.

It should be noted that the use of faciograms in stratigraphic investigations and structural geological survey work demands a special attention of the investigator to the sections under consideration. It is necessary to register accurately all the facts of interbedding of some or other sediments with the thicknesses, the number of interbeddings; to count the number of bands and to note their distribution within the section of the stratum. All of these facts, showing the character of bottom movements, may greatly assist the work of correlation of the sections and to reveal the mechanism of synchronous movements of the Earth's crust.

Thus, by means of faciograms, as additional constructions in the treatment of the results of different geological, regional and detailed investigations, as well as in prospecting for sedimentary useful minerals (coal, oil, salts, sedimentary deposits of metals, phosphorites, etc.), we get the possitility of a deeper insight into the geological processes of the past. Making use of such faciograms, the author found it possible frequently to obtain new interesting data, and on the basis of these practically and scientifically important deductions often arose quite unexpectedly.

Radium Institute
Academy of Sciences of the USSR.

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SOIL SCIENCE

A METHOD FOR DETERMINING THE ABSOLUTE AGE OF SOILS

By I. I. SINIAGUIN

(Communicated by L. I. Prasolov, Member of the Academy, 20. I. 1943)

Among the problems of soil science one of the central places is occupied by the problem of determining the absolute age of soils. This problem is one of the least developed. Two methods of attacking it, different in principle, are possible. First; an approximate idea of the absolute age of soils may be derived from the history of a given surface. Secondly; to this end use may be made of the peculiarities of the composition of soils according to genetic horizons, relating them to the probable rates of migration of some substance or other under the influence both of physical-chemical and of biological factors. Our aim is to propose a new method of approximate determination of the absolute age of soils; based on the biological accumulation of phosphoric acid in the upper horizons.

As is known, the phosphoric acid assimilated by plants both from the upper and from the subjacent horizons after the dying away of the overground part and roots (on a virgin land) is fixed in the upper horizon. The slight solubility of soil phosphates and their highly intensive fixation in a biological way in plants and micro-organisms causes a complete or nearly complete absence of their migration down the profile. Hence, knowing the annual amount of phosphates; accumulation in the upper horizon as a result of their transfer by plants from the lower horizons, as well as the difference in the P_2O_5 content in the upper layer and in the matrix unaltered by soil formation, we can approximately determine the absolute age of a given soil.

It is from the very principle of the proposed method that it follows that it is not applicable to all soils. The necessary conditions for applying this method are: 1) Uniformity of the initial bulk chemical composition of the rock, from which the soil has developed, to a sufficiently great depth. 2) Development of the soil all the time under the same external conditions; in the first place with a more or less constant composition of vegetation and in a uniform climatic environment. 3) Absence of erosion and deposition. 4) Absence of a very strong shifting of soil masses from some horizons to other due to the soil digging activity of special animals.

When the above conditions are satisfied, the correctness of the principle of the proposed method may hardly leave any room for doubt. However, some difficulties in its application are caused by the insufficient development of the problem of phosphate migration in the soil under the influence of physical-chemical factors, as well as of the question of the degree of utilization of phosphoric acid by various vegetable associations from different horizons of the soil. With the present status of our knowledge, we have as yet no satisfactory methods which would enable us to solve these problems in a quite reliable manner experimentally. Now the following provisional assumptions seem to be the most correct.

1) As shown by numerous agrochemical investigations, in the vast majority of soils the downward washing, even of the comparatively mobile phosphates

of mineral fertilizers; is lacking entirely or nearly so. Therefore we are justified in assuming that the washing of phosphates from the upper layer 15—30 cm thick is generally so slightly pronounced that it may be practically neglected.

2) It may be assumed that plants assimilate phosphoric acid from different horizons roughly proportionately to the «mobility» («assimilability») of P_2O_5 within them (this being determined by one of the existing agrochemical methods) and proportionately to the mass of the active root system in these horizons.

Using the proposed method, we have made an attempt to determine the absolute age of a typical serozem (Arys region, South Kazakhstan, about 1.5 km NE of the Kalash station, Turk.-Sib. railway). The soil investigated is developed on a homogeneous thick loess. The structure of the profile is a common one for typical serozems, repeatedly described in the literature. The ground waters occur at a considerable depth, in any case deeper than 10 m. The surface of the selected plot is even, its lay excluding both the possibility of erosional processes and a supply of particles from outside. According to data found in literature ([1],[2]), the climatic conditions in the present serozem zone were distinguished by a relative persistence for a prolonged time, without proceeding beyond an arid and extra-arid climate characteristic of this zone at present. The relative constancy of the climatic conditions suggests a certain uniformity of the type of vegetation during a sufficiently long period of time. Thus, also from this standpoint the object selected seems to conform to the method proposed. To-day the vegetation is represented here mainly by *Ephemera*; predominant plant is *Poa bulbosa*; *Holdeum murinum* and other small *Ephemera* are found in a lesser quantity; there are found isolated specimens of *Ferula* sp.; *Eremurus spectabilis*, etc. The field part of the investigation was carried out in the first decade of May, i. e. during the period of the greatest development of *Ephemera*.

An estimation of the earth mass and root system of plants was made on 1-m lots, being repeated twelve times over, the following results were obtained:

	Air dry mass of plants, g/m²	P_2O_5 content, %	P_2O_5 content g/m² in the mass
Overground green mass	135.58 ± 14.30	0.185 ± 0.011	0.251
Sod and roots in layer 0—20 cm	954.54 ± 10.36	0.164 ± 0.021	1.565
Including roots	222.38 ± 31.06	—	—
Roots in layer 20—120 cm	65.9 ± 6.72	—	—
Ratio of root mass in a 20—120 cm layer to their total mass	0.229	—	—

Determinations of «mobile» P_2O_5 (coal-ammoniacal extracts) have shown that in horizon 0—20 the latter is contained in an amount of 42.08, and in horizon 20—120 (in the average sample) in an amount of 5.08 mg to 1000 cm³ of soil. From the above, the annual removal of P_2O_5 from horizon 20—120 cm to horizon 0—20 cm has been determined by us as 0.0439 g per area of 1 m².

The bulk chemical analysis of soil according to horizons and a recalculation of the figures obtained to a substance containing no carbonates, no humus, no water and no salts have shown that in horizon 0—20 cm 0.228% P_2O_5 is contained in the rock; in layer 120—150 cm, 0.158%. Upon recalculating the above figures after Kossovich([3]) (Fe_2O_3 being taken as the oxide-index) and estimating the volume weight of the soil, it was found that in layer 0—20 cm 182.5 g P_2O_5 have been accumulated as compared to the rock. From this the absolute age of soil has been determined, which is 182.5 : 0.0439=4157, or roughly 4000 years.

All-Union Institute of Beet Culture.
Alma-Ata Branch.

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